

Comparative Analysis of Structurally Different Dynamical Systems

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Abstract

CEC* is an implemented method for comparative analysis (CA) which is able to deal with intra-model and inter-model CA problems within a single framework. The algorithm constructs a comparative envisionment showing (1) possible effects of differences in the antecedent conditions or (2) possible causes of differences in the response of the two systems. CEC* has been proven sound and incomplete.

1 Introduction

Qualitative reasoning (QR) is concerned with predicting and explaining the qualitative behavior of dynamical systems. An important QR problem is the comparison of behaviors of two systems, that is, an assessment of the relative value of system variables at chosen time-points. More specifically, this *comparative analysis (CA)* task is directed at (1) predicting possible consequences of differential antecedent conditions and (2) finding possible causes of differential responses of the two systems.

CA questions arise in a variety of contexts. Consider an engineer diagnosing an electrical circuit by comparing the observed faulty behavior with a reference behavior. Or scientists estimating the impact of a disturbing process on a quantity measured in an experiment.

With few exceptions, QR methods for comparative analysis have focussed on the comparison of systems with the same structure: *intra-model CA* [Weld, 1990; Chiu and Kuipers, 1992; Neitzke and Neumann, 1994; de Jong *et al.*, 1996]. The comparison of dynamical systems with a different structure, *inter-model CA*, is considered to be a "terribly difficult" problem in general [Weld, 1990]. The limitation to intra-model CA is a serious shortcoming, since many interesting questions can be mapped onto inter-model CA problems, including the examples given above.

In this paper we introduce CEC*, an implemented method for comparative analysis which treats intra-model CA and inter-model CA within a single framework [de Jong, 1996]. The exposition starts with the definition of some basic concepts and a formulation of

the method as a propagation process (sec. 2). The constraints employed in the propagation process form the topic of sec. 3. Sec. 4 shows how CEC* deals with an example of a predictive and a diagnostic CA question. The discussion in sec. 5 gives correctness guarantees and pays attention to the generality of the method. Brief notes on further research conclude the paper.

2 Basic concepts and outline of method

2.1 Pairs of comparison

In a comparative analysis a qualitative behavior of a first system is compared with a behavior of a second system. We assume that the qualitative behaviors have been generated by the QSIM algorithm [Kuipers, 1994], although the principles underlying our method do not depend on this choice.

Fig. 1 shows a system of two cascaded tanks. The (constant) input variables are the inflow i and the sizes r_u, r_l of the orifices of the upper and lower tank. The amounts of water a_u, a_l in the tanks form the state variables. One of the qualitative behaviors produced by QSIM for the cascaded tanks is summarized in fig. 1(c). As can be seen, the upper tank reaches equilibrium before the lower tank.

Now suppose we compare this system with a second cascaded-tanks system that has a *leaky* upper tank. The leaky system is structurally different from the watertight one: the netflow into the upper tank is given by $\hat{n}_u = \hat{i} - \hat{o}_u - \hat{o}_h$, where $\hat{o}_h = h(\hat{a}_u)$, $\partial h / \partial \hat{a}_u > 0$.¹ The qualitative behavior in which the upper tank reaches equilibrium first is also a behavior of the leaky system.

We will compare the watertight and leaky cascaded tanks when both exhibit the behavior of fig. 1(c). The behaviors of the two systems are topologically equal [Weld, 1990] with respect to the shared variables (a situation not obtaining in general, see sec. 4). The two behaviors are compared at so-called *pairs of comparison*. A pair of comparison pc is a pair of distinguished time-points $\langle t, \hat{t} \rangle$ from the first and the second behavior, respectively.

¹Following [Weld, 1990], the hat accent $\hat{\cdot}$ will be used to denote the variables of the second system.

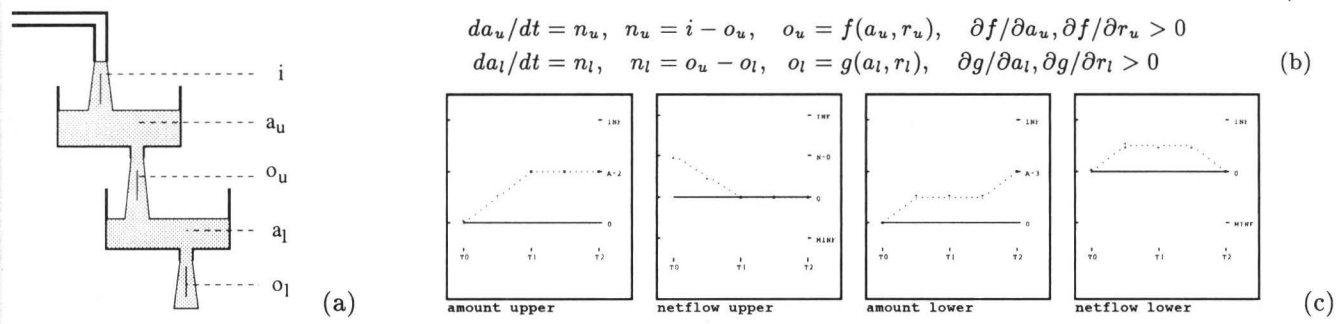


Figure 1: Qualitative description of the structure and behavior of a watertight cascaded-tanks system (adapted from [Kuipers, 1994]). The variables have the following interpretation: a water amount, r size of orifice, o outflow, i inflow, and n netflow. The subscripts u , l , and h refer to the upper tank, lower tank, and leak, respectively. The system (a) has the qualitative differential equation (QDE) (b). A qualitative behavior of the system is shown in (c).

Def. 1 (Meaningful pairs of comparison) Two behaviors are given with sequences of distinguished time-points $t_0, \dots, t_n$ and $\hat{t}_0, \dots, \hat{t}_m$. The pair of comparison $pc = \langle t, \hat{t} \rangle$ is a *meaningful pair of comparison*, if

1. t and $\hat{t}$ are the initial time-points t_0 and $\hat{t}_0$ or the final time-points t_n and $\hat{t}_m$;
2. t and $\hat{t}$ are time-points at which variables p and $\hat{p}$ reach the same basic landmark value $0, \infty$, or $-\infty$.

A particular pair of comparison could be singled out by both of the criteria in the definition. For the behaviors of the watertight vs. the leaky cascaded-tanks system we find three meaningful pairs of comparison: $pc_0 = \langle t_0, \hat{t}_0 \rangle$, $pc_1 = \langle t_1, \hat{t}_1 \rangle$, $pc_2 = \langle t_2, \hat{t}_2 \rangle$ (see fig. 1(c)).

A partial ordering relation can be defined on the pairs of comparison which formalizes the intuitive notion that one pair of comparison occurs before another. For two pairs of comparison $pc_1 = \langle t_1, \hat{t}_1 \rangle$ and $pc_2 = \langle t_2, \hat{t}_2 \rangle$ it holds that $pc_1 \preceq pc_2$, iff $t_1 \leq t_2$ and $\hat{t}_1 \leq \hat{t}_2$. If pc_1 and pc_2 are different pairs of comparison and $pc_1 \preceq pc_2$, then pc_1 is said to be a *predecessor* of pc_2 (and pc_2 a *successor* of pc_1). A set of pairs of comparison ordered according to the $\preceq$ -relation will be called an *ordered pairs of comparison (OPC) structure*. This structure always has a top element, the first pair of comparison, and a bottom element, the last pair of comparison. In the example we have the simple OPC structure $pc_0 \preceq pc_1 \preceq pc_2$.

Two successive pairs of comparison in an OPC structure mark out what we will call a pair of *behavior fragments*, one for each system. The behavior fragments defined by the successive pairs of comparison $pc_1 = \langle t_1, \hat{t}_1 \rangle$ and $pc_2 = \langle t_2, \hat{t}_2 \rangle$ are the closed intervals $[t_1, t_2]$ and $[\hat{t}_1, \hat{t}_2]$. If the interval of a behavior fragment does not contain any distinguished time-points except for its boundaries, we will say it is a *primitive behavior fragment*, otherwise a *composite behavior fragment*. Combining the three pairs of comparison identified for the cascaded-tanks example with the qualitative behavior in fig. 1(c), we directly see that all behavior fragments $([t_0, t_1], [t_1, t_2], [\hat{t}_0, \hat{t}_1], [\hat{t}_1, \hat{t}_2])$ are primitive.

2.2 Relative values

Pairs of comparison have been introduced to specify when the values of variables in the two systems can be meaningfully compared. In order to express how the values of a shared variable p in the first and second system relate at a pair of comparison, the notion of *relative value (RV)* is introduced.

Def. 2 (Relative value) Given a variable p and a pair of comparison $pc = \langle t, \hat{t} \rangle$. The relative value (RV) at pc is defined as:

$$\begin{aligned} p \uparrow_{pc}, & \text{ iff } \hat{p}(\hat{t}) > p(t), \\ p \parallel_{pc}, & \text{ iff } \hat{p}(\hat{t}) = p(t), \\ p \downarrow_{pc}, & \text{ iff } \hat{p}(\hat{t}) < p(t). \end{aligned}$$

Often the RV of a variable at a pair of comparison pc is denoted by $RV(p)_{pc}$, and $RV(p)_{pc} = \uparrow$ is equivalent to $p \uparrow_{pc}$. Remark that a relative value is only defined for variables that are shared by the two systems. A special kind of RV is the relative duration of the behavior fragments defined by two successive pairs of comparison. Written as $RV(T)_{pc_1 \rightarrow pc_2}$, this RV expresses whether the intervening behavior fragment of the second system has a longer, shorter, or equal duration compared to that of the first system.

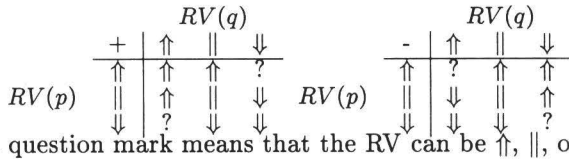
A few basic (in)equality relations and algebraic operations for RVs can be introduced. This simple algebra is used in the definition and evaluation of propagation constraints for comparative analysis that are discussed below.

Def. 3 (Equality and inequality) For two relative values the equality $RV(p) = RV(q)$ and inequality $RV(p) < RV(q)$ relations are defined as follows:

$$\begin{array}{c|ccc} & \uparrow & \parallel & \downarrow \\ \hline RV(p) & \begin{array}{c} \uparrow \\ \parallel \\ \downarrow \end{array} & \begin{array}{c} T \\ F \\ F \end{array} & \begin{array}{c} F \\ T \\ T \end{array} \\ & & RV(p) & \end{array} \quad \begin{array}{c|ccc} & \uparrow & \parallel & \downarrow \\ \hline & \begin{array}{c} \uparrow \\ \parallel \\ \downarrow \end{array} & \begin{array}{c} T \\ T \\ T \end{array} & \begin{array}{c} F \\ F \\ F \end{array} \\ & & RV(q) & \end{array}$$

Similar definitions can be given for $>$, $\geq$, $\leq$, and $\geq$.

Def. 4 (Basic algebraic operations) For two relative values the addition $RV(p) + RV(q)$ and subtraction $RV(p) - RV(q)$ operations are defined as follows:



A question mark means that the RV can be $\uparrow$, $\parallel$, or $\downarrow$.

By means of the concepts introduced above comparative analysis can be formalized as a propagation problem. The input of CEC* consists of a pair of qualitative behaviors, the QDEs giving rise to them, and a set of relative values at either the first or the last pair of comparison, that is, the top or the bottom of the OPC structure. The analysis proceeds by propagating the initial RVs through the OPC structure, from each pair of comparison to its successor(s) or predecessor(s). When the direction of propagation is from top to bottom, the method is *predicting* the consequences of differential antecedent conditions. The reverse direction corresponds with the *explanation* or *diagnosis* of differential responses of the two systems.

For this propagation process to succeed we need *propagation* or *RV constraints*. These constraints relate RVs at the same pair of comparison or at successive pairs of comparison.

3 Constraints on relative values

3.1 Variables at a pair of comparison

A direct way to infer the RV of a variable p at a pair of comparison $pc = \langle t, \hat{t} \rangle$ is to examine the qualitative value of the continuously differentiable functions p and $\hat{p}$ at t and $\hat{t}$. The qualitative value of a variable $p(t)$ is either a landmark value l_j or an open interval between two landmark values $[l_j, l_{j+1}]$ in which $p(t)$ falls. If we extend the applicability of $>$, $=$, and $<$ to intervals, the following result is obtained:²

Prop. 1 (Qualitative values) Let p_v and $\hat{p}_v$ be qualitative values of p and $\hat{p}$, respectively, at pair of comparison $pc = \langle t, \hat{t} \rangle$. For the difference $\hat{p}(\hat{t}) - p(t)$ we have:

$\hat{p}(\hat{t}) - p(t)$		> 0	$= 0$	< 0
$\hat{p}_v$	> 0	$?$	> 0	> 0
	$= 0$	< 0	$= 0$	> 0
	< 0	< 0	< 0	$?$

Knowing the sign of the difference $\hat{p}(\hat{t}) - p(t)$ immediately gives the relative value of p at pc , $RV(p)_{pc}$. For example, in fig. 1(c) we see that at pc_0 $a_u = 0$ and $\hat{a}_u = 0$, so that $RV(a_u)_{pc_0} = \parallel$.

3.2 Functions at a pair of comparison

Suppose that a variable p shared by the two systems is a continuously differentiable function of a vector of variables $\mathbf{r}$ in the first system and $\mathbf{s}$ in the second system:

$$p = f(\mathbf{r}) \quad \text{and} \quad \hat{p} = g(\hat{\mathbf{s}}) \quad (1)$$

The problem is that $p(t)$ and $\hat{p}(\hat{t})$ cannot be straightforwardly compared, because in general the vectors $\mathbf{r}$ and

$\mathbf{s}$ and the functions f and g will be different. To make the expressions for p and $\hat{p}$ comparable, define $\mathbf{q}$ as the vector of variables occurring both in $\mathbf{r}$ and in $\mathbf{s}$, and $\mathbf{a}$ as the vector of variables occurring either in $\mathbf{r}$ or in $\mathbf{s}$, but not in both. We can now give a criterion for the comparability of p and $\hat{p}$:

Def. 5 (Comparable functions) Suppose p and $\hat{p}$ are defined by continuously differentiable functions f and g , as in (1). The functions f and g are called *comparable* with respect to a continuously differentiable function h and vectors $\mathbf{a}^0$ and $\hat{\mathbf{a}}^0$, if

$$p = h(\mathbf{q}, \mathbf{a}^0) \quad \text{and} \quad \hat{p} = h(\hat{\mathbf{q}}, \hat{\mathbf{a}}^0)$$

The variable a_i^0 ($\hat{a}_i^0$) equals either a variable in $\mathbf{r}$ ($\hat{\mathbf{s}}$) or, if not occurring in $\mathbf{r}$ ($\hat{\mathbf{s}}$), has a *comparison value* c_i ($\hat{c}_i$).

An example will clarify this definition. From the cascaded-tanks QDEs we take $n_u = i - o_u$ and $\hat{n}_u = \hat{i} - \hat{o}_u - \hat{o}_h$. The vector $\mathbf{q}$ of shared variables is $[i \ o_u]'$ and $\mathbf{a} = o_h$. Now let $n_u = h([i \ o_u]', 0) = i - o_u$ and $\hat{n}_u = h([\hat{i} \ \hat{o}_u]', \hat{o}_h) = \hat{i} - \hat{o}_u - \hat{o}_h$, then h makes f and g comparable. The comparison value 0 is chosen for the flow out of the leak in the first system, i.e. $o_h = 0$.

With the generalized mean value theorem for differentiation we find:

Prop. 2 (Functional relations) The variables p and $\hat{p}$ are defined by functions f and g . If f and g are comparable with respect to a function h , then at pair of comparison $pc = \langle t, \hat{t} \rangle$:

$$\hat{p}(\hat{t}) - p(t) = \mathbf{d}_q(\hat{\mathbf{q}}(\hat{t}) - \mathbf{q}(t)) - \mathbf{d}_a(\hat{\mathbf{a}}^0(\hat{t}) - \mathbf{a}^0(t))$$

$\mathbf{d}_q$ and $\mathbf{d}_a$ are vectors with $d_{q,i} = \frac{\partial}{\partial q_i} h(\psi, \mathbf{a}^0)$ and $d_{a,i} = \frac{\partial}{\partial a_i} h(\psi, \mathbf{a}^0)$. Each ψ_j lies between $q_j(t)$ and $\hat{q}_j(\hat{t})$, and each α_j^0 between $a_j^0(t)$ and $\hat{a}_j^0(\hat{t})$.

In the cascaded-tanks example we have at pc_0

$$\begin{aligned} \hat{n}_u(\hat{t}_0) - n_u(t_0) &= (\hat{i}(\hat{t}_0) - i(t_0)) \\ &\quad - (\hat{o}_u(\hat{t}_0) - o_u(t_0)) - (\hat{o}_h(\hat{t}_0) - 0) \end{aligned}$$

Since the tanks are filled from empty, $\hat{o}_h(\hat{t}_0) = 0$ and the last term disappears. The expression can be easily transformed into an RV constraint:

$$RV(n_u)_{pc_0} = RV(i)_{pc_0} - RV(o_u)_{pc_0}$$

Suppose that the inflow and outflow are known to be equal in the two systems ($i|_{pc_0}, o_u|_{pc_0}$), then we can readily infer with defs. 3 and 4 that the net flow is also equal ($n_u|_{pc_0}$).

Prop. 2 is equally applicable to intra-model and inter-model CA problems. In the former case, the qualitative models of the two systems, and thus all corresponding functional relations, are identical. Consequently, $\mathbf{a} = 0$ and $\hat{p}(\hat{t}) - p(t) = \mathbf{d}_q(\hat{\mathbf{q}}(\hat{t}) - \mathbf{q}(t))$.

²For proofs of the propositions, see [de Jong, 1996].

3.3 State variables between pairs of comparison

The question to be addressed now is how the relative values of variables at a pair of comparison $pc_1 = \langle t_1, \hat{t}_1 \rangle$ relate to those at a predecessor pair of comparison $pc_0 = \langle t_0, \hat{t}_0 \rangle$. For this, we have to take into account how the variables of the two systems change during the respective behavior fragments between the pairs of comparison.

We will assume that the two behavior fragments start simultaneously, i.e. $t_0 = \hat{t}_0$. If this is not the case, the behavior fragments have to be synchronized first by a suitable time translation. $T = t_1 - t_0$ and $\hat{T} = \hat{t}_1 - \hat{t}_0$ represent the durations of the two behavior fragments.

We can distinguish several cases in the comparison of two behavior fragments by considering how their durations relate. When comparing primitive behavior fragments, we have $\hat{T} < T$, $\hat{T} = T$, or $\hat{T} > T$, and we can rewrite the difference in x at pc_1 as follows:

$$\hat{x}(\hat{t}_1) - x(t_1) = \begin{cases} x(\hat{t}_1) - x(t_1) + \hat{x}(\hat{t}_1) - x(\hat{t}_1) & , \hat{T} < T \\ \hat{x}(\hat{t}_1) - x(\hat{t}_1) & , \hat{T} = T \\ \hat{x}(\hat{t}_1) - \hat{x}(t_1) + \hat{x}(t_1) - x(t_1) & , \hat{T} > T \end{cases}$$

Consider the case $\hat{T} < T$. The difference $\hat{x}(\hat{t}_1) - x(t_1)$ is split up into $x(\hat{t}_1) - x(t_1)$ and $\hat{x}(\hat{t}_1) - x(\hat{t}_1)$. The sign of the former part directly follows from the qualitative behavior of x , whereas the latter part must be determined.

For the general problem of establishing the difference $\hat{x}(t_b) - x(t_b)$ in terms of the difference of variables at t_a , we have to study the QDEs which define the dynamical behavior of the two systems. First, we introduce three classes of variables: shared state variables x , shared input variables u , and auxiliary variables a . A variable v is (1) a *shared state* variable if it is shared and a state variable in at least one of the systems, (2) a *shared input* variable if it is shared and an input variable in at least one of the systems, or (3) an *auxiliary* variable if it is not shared. The QDEs of the two systems give rise to the state equations:

$$\frac{dx(t)}{dt} = f_1(x(t), u(t), a(t)), \quad \frac{d\hat{x}(t)}{dt} = f_2(\hat{x}(t), \hat{u}(t), \hat{a}(t)) \quad (2)$$

We can now define a *comparison system* and transform it into a familiar linear form:

Prop. 3 (Linear comparison system) Consider the comparison system obtained by subtracting the state equations (2). If the functions f_1 and f_2 are comparable through a function h , such that $f_1(x, u, a) = h(x, u, a^0)$ and $f_2(\hat{x}, \hat{u}, \hat{a}) = h(\hat{x}, \hat{u}, \hat{a}^0)$, then we can write the state equation of the comparison system as:

$$\frac{d}{dt}(\hat{x}(t) - x(t)) = A(t)(\hat{x}(t) - x(t)) + B(t)(\hat{u}(t) - u(t)) + E(t)(\hat{a}^0(t) - a^0(t)) \quad (3)$$

$A(t)$, $B(t)$, and $E(t)$ are time-varying matrices of continuous functions $a_{ij}(t) = \frac{\partial}{\partial x_j} h_i(\xi(t), \nu(t), \alpha^0(t))$,

$$b_{ik}(t) = \frac{\partial}{\partial u_k} h_i(\xi(t), \nu(t), \alpha^0(t)), \quad \text{and} \\ e_{il}(t) = \frac{\partial}{\partial a_l} h_i(\xi(t), \nu(t), \alpha^0(t)). \quad \xi_j(t) \text{ lies between } \hat{x}_j(t) \text{ and } x_j(t), \nu_k(t) \text{ between } \hat{u}_k(t) \text{ and } u_k(t), \text{ and } \alpha_l^0(t) \text{ between } \hat{a}_l^0(t) \text{ and } a_l^0(t).$$

The proposition follows directly with the definition of comparable functions and the application of prop. 2 to each h_i .

For the comparison of watertight and leaky cascaded tanks, the relevant vectors of variables are $x = [a_u \ a_l]'$, $u = [i \ r_u \ r_l]'$, and $a = o_h$. With the QDE in fig. 1 we determine the following matrices in state equation (3):

$$A(t) = \begin{bmatrix} -a_{11}(t) & 0 \\ a_{21}(t) & a_{22}(t) \end{bmatrix}, \quad E(t) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \\ B(t) = \begin{bmatrix} 1 & -b_{12}(t) & 0 \\ 0 & b_{22}(t) & -b_{23}(t) \end{bmatrix}, \quad (4)$$

where the matrix elements are given by the partial derivatives $\partial h_i / \partial x_j$, $\partial h_i / \partial u_k$, and $\partial h_i / \partial a_l$. When considering the comparison system on an interval in which $x, \hat{x}, u, \hat{u}, a^0, \hat{a}^0$ have signs as between pc_0 and pc_1 , we find that $a_{ij}, b_{ij} > 0$.

Again, it is useful to point out how intra-model CA is treated as a special case of inter-model CA. When comparing two structurally identical systems, the term $E(t)(\hat{a}^0(t) - a^0(t))$ drops out of (3).

The formulation of a linear comparison system is a pivotal move in deriving RV constraints between successive pairs of comparison. An explicit solution to (3) is a basic result in linear system theory [Rugh, 1996].

Prop. 4 (State variables) Suppose two primitive behavior fragments are compared. The difference between the shared state variables x at $pc_1 = \langle t_1, \hat{t}_1 \rangle$ can be expressed in terms of differences at $pc_0 = \langle t_0, \hat{t}_0 \rangle$ as:

$$\hat{x}(\hat{t}_1) - x(t_1) = \begin{cases} x(\hat{t}_1) - x(t_1) + F(\hat{t}_1, t_0) & , \hat{T} < T \\ F(t_1, t_0) & , \hat{T} = T \\ \hat{x}(\hat{t}_1) - \hat{x}(t_1) + F(t_1, t_0) & , \hat{T} > T \end{cases} \quad (5)$$

where the function F is defined as follows:

$$F(t_b, t_a) = \Phi(t_b, t_a)(\hat{x}(t_a) - x(t_a)) + \int_{t_a}^{t_b} \Phi(t_b, \tau) [B(\tau)(\hat{u}(t_a) - u(t_a)) + E(\tau)(\hat{a}^0(\tau) - a^0(\tau))] d\tau \quad (6)$$

The usefulness of prop. 4 hinges on the possibility to determine the transition matrix $\Phi(t_b, \tau)$ for all τ on an interval $[t_a, t_b]$. Explicitly solving for $\Phi(t_b, \tau)$ is generally a very difficult task, but for our purposes it suffices to determine the *signs* of the elements of $\Phi(t_b, \tau)$. A direct way to find such a qualitative transition matrix is to trace the zero-input solutions of (3) in the phase space for initial states consisting of the column vectors of the unit matrix I (see, e.g., [Rugh, 1996]). Here we restrict ourselves to stating the resulting transition matrix for the cascaded tanks between pc_0 and pc_1 :

$$\Phi(t_b, \tau) = \begin{bmatrix} \phi_{11}(t_b, \tau) & 0 \\ \phi_{21}(t_b, \tau) & \phi_{22}(t_b, \tau) \end{bmatrix},$$

with $\phi_{ij}(t_b, \tau) > 0$ for all τ in $[t_a, t_b]$.

Armed with the vectors $\mathbf{x}$, $\mathbf{u}$, $\mathbf{a}^0$ and matrices $\mathbf{B}$, $\mathbf{E}$, Φ determined above, we can use prop. 4 to express differences at pc_1 in terms of differences at pc_0 . Evaluating (5) for $\hat{T} < T$, we obtain for state variable a_u :

$$\hat{a}_u(\hat{t}_1) - a_u(t_1) = a_u(\hat{t}_1) - a_u(t_1) + c_1(\hat{a}_u(\hat{t}_0) - a_u(t_0)) \\ + c_2(\hat{i}(\hat{t}_0) - i(t_0)) - c_3(\hat{r}_u(\hat{t}_0) - r_u(t_0)) - c_4 \quad (7)$$

and $c_1, \dots, c_4 > 0$. The constant factors are transition matrix elements (e.g., $c_1 = \phi_{11}(\hat{t}_1, t_0)$) or integrals involving transition matrix elements (e.g., $c_3 = \int_{t_0}^{\hat{t}_1} \phi_{11}(\hat{t}_1, \tau) b_{12}(\tau) d\tau$). From the qualitative behavior in fig. 1(c) we see that the amount of water in the upper tank is increasing between pc_0 and pc_1 , so $a_u(\hat{t}_1) - a_u(t_1) < 0$ (since $\hat{t}_1 < t_1$).

Eq. (7) can be directly translated into a constraint:

if $RV(T)_{pc_0 \rightarrow pc_1} = \Downarrow$, then

$$RV(a_u)_{pc_1} < RV(a_u)_{pc_0} + RV(i)_{pc_0} - RV(r_u)_{pc_0}$$

The RV constraint implies, by contraposition, that from $a_u \parallel_{pc_0}$, $a_u \parallel_{pc_1}$, $i \parallel_{pc_0}$, $r_u \parallel_{pc_0}$ we must conclude that the relative duration from pc_0 to pc_1 cannot be shorter.

3.4 Constants between pairs of comparison

Besides state variables, constants also have their difference propagated between pairs of comparison.

Prop. 5 (Constants) Suppose two behavior fragments defined by $pc_0 = \langle t_0, \hat{t}_0 \rangle$ and $pc_1 = \langle t_1, \hat{t}_1 \rangle$ are compared. If the variable c has a constant value during the behavior fragments, then $\hat{c}(\hat{t}_1) - c(t_1) = \hat{c}(\hat{t}_0) - c(t_0)$.

The constancy of the inflow in the example leads to the constraint $RV(i)_{pc_1} = RV(i)_{pc_0}$.

4 Results

The CEC* algorithm outlined in sec. 2 has been implemented in Common Lisp, as a module interacting with the QSIM program. At each pair of comparison a constraint satisfaction problem is generated consisting of the variables shared by the two systems, domains $\{\Downarrow, \Uparrow, \parallel\}$ of their possible relative values, and constraints relating the RVs of variables at the current and at predecessor pairs of comparison. The RV constraints are instantiated from a database with templates derived from props. 1, 2, 4, 5. A consistent combination of RVs for the variables at a pair of comparison is called a *comparative state*. There may be several comparative states at a particular pair of comparison. The comparative states at a pair of comparison are linked to comparative states of predecessor and successor pairs of comparison to form a *comparative envisionment* [de Jong *et al.*, 1996]. Branching in the comparative envisionment is possible, because at the pairs of comparison an RV is either $\Downarrow$, $\parallel$, or $\Uparrow$.

An example of a comparative envisionment produced by CEC* is shown in fig. 2(a). The envisionment answers the predictive CA question: How does the equilibrium

state of a cascaded-tanks system with a leaky upper tank differ from that with a watertight upper tank when the systems are considered under the same conditions? Each path from the first to the last comparative state in the envisionment forms a *comparative behavior*. Although ambiguities arise with respect to a_i, n_i at pc_1 , the five possible comparative behaviors predict a single outcome: the leak tends to lower the amount of water in the tanks at equilibrium ($CS_{pc_2, 0}$). No unambiguous conclusion can be drawn about the relative durations to reach the equilibrium state.

Fig. 2(b) illustrates a diagnostic CA question: When the relative amounts of water at pc_1 and pc_2 have been measured for watertight and leaky cascaded tanks filled from empty, and the orifices have the same size, which differences could account for the lower equilibrium water amounts in the leaky system? CEC* finds three possible causes: (1) a leaky upper tank and lower inflow ($CS_{pc_0, 0}$), (2) a leaky upper tank and higher inflow ($CS_{pc_0, 2}$), and (3) a leaky upper tank only ($CS_{pc_0, 1}$). The leak, making the second system structurally different from the first, tends to lower the equilibrium amounts at pc_2 . An additional difference in initial conditions may strengthen ($i \Downarrow_{pc_0}$) or attenuate ($i \Uparrow_{pc_0}$) this effect.

The CEC* program has been tested on a range of intra-model and inter-model CA questions concerning simple and more advanced QR systems, such as bathtubs, watertanks filled by a pump, masses on a spring, heat exchangers, sliding blocks, and tensile fracture tests. These examples generalize upon the cascaded-tanks example by requiring CEC* to deal with topologically different qualitative behaviors, region transitions, and composite behavior fragments between pairs of comparison. We are currently embedding CEC* in a system for the integration of measurements from a scientific knowledge base.

5 Discussion and related work

The CEC* algorithm has been proven sound and incomplete [de Jong, 1996]. Analogously to [Kuipers, 1994], soundness is taken to mean that for any two real behaviors of the systems that are in conformity with CEC*'s input, a comparative behavior describing their differences is included in the comparative envisionment. On the other hand, a comparative envisionment may contain comparative behaviors that do not correspond to any two real behaviors in conformity with CEC*'s input. CEC* then fails to derive a stronger conclusion and is consequently incomplete.

The proof of soundness is enabled by the mathematical foundation of the RV constraints. The propositions in sec. 3 also help in judging the generality of the method. In principle, the theorems apply to systems of arbitrary order and structure. A practical problem in evaluating (5) of prop. 4, however, is that it may be difficult to establish whether a transition matrix function $\phi_{ij}(t_b, \tau)$ changes sign during $[t_a, t_b]$, and hence to establish the sign of the elements of $\Phi(t_b, t_a)$ and $\int_{t_a}^{t_b} \Phi(t_b, \tau) \dots d\tau$ in

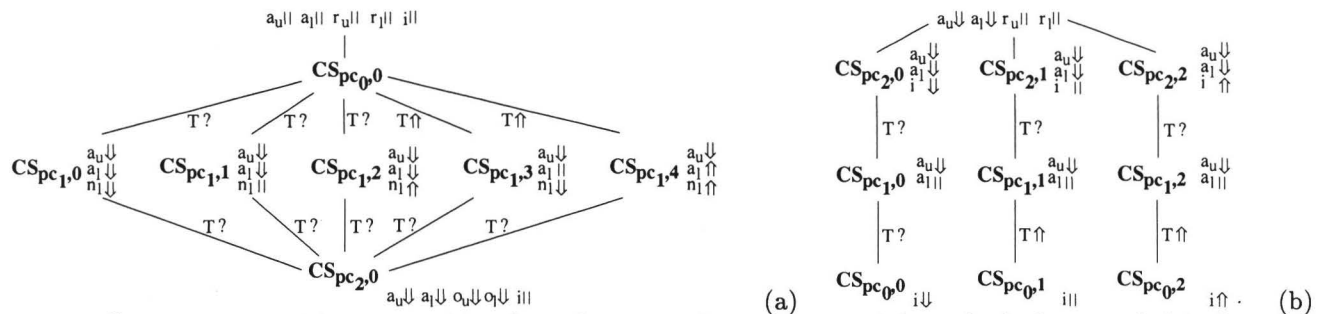


Figure 2: Comparative envisionment arising from the comparison of a watertight and a leaky cascaded-tanks system when both show the qualitative behavior of fig. 1(c). Comparative envisionment (a) is produced in response to a predictive CA question, whereas (b) arises from a diagnostic CA question. A few distinctive RVs are indicated at the comparative states (*CS*). The links between comparative states are labelled with the relative durations.

$F(t_b, t_a)$. The formulation of the difficulty in terms of concepts from linear system theory has the advantage of making this problem well-defined and of suggesting possible solutions (see [de Jong, 1996]).

The computational complexity of CEC* is of the order $\mathcal{O}(n3^q)$, where q is the (maximum) number of shared variables and n the number of pairs of comparison in the OPC structure. In the examples we have studied, the average-case behavior turned out to be more agreeable and could be improved by integrating additional RV constraints (e.g., measurements of variables at a pair of comparison) during the analysis.

CEC* borrows ideas on the mathematical formulation and the propagation approach from existing CA methods, in particular DQ analysis [Weld, 1990] and CEC [de Jong *et al.*, 1996]. A notable exception to the rule that existing CA methods are restricted to intra-model CA is the model sensitivity approach described in [Weld, 1992].³ In this approach, a behavior arising from a simple model is compared with a behavior from a more complex model. The simple model must be a *fitting approximation* of the more complex model, that is, the complex model must have an independent variable, such that, when this parameter is taken to a certain approximation limit, the RVs of the shared variables become $\parallel$. By constructing fitting approximations, the *inter-model* CA problem can be reduced to an *intra-model* CA problem.

The method's applicability thus depends on the opportunity to formulate a fitting approximation, which is not always possible (e.g., in the cascaded-tanks example). In addition, the implementation of the model sensitivity approach solves the resulting intra-model CA problem by DQ analysis. Unlike CEC*, DQ analysis terminates when confronted with ambiguities and can only reason in one direction through the OPC structure.

6 Conclusions

CEC* is an implemented method for predictive and diagnostic comparative analysis that is able to deal with

³RSIM+ [Neitzke and Neumann, 1994] allows for a modest inter-model CA feature, namely differences in the monotonicity of functional relations.

intra-model and inter-model problems within a single framework. The constraints it uses in RV propagation have a solid mathematical foundation. This makes it possible to proof CEC*'s soundness and incompleteness.

Further work on CEC* concerns improvements upon the derivation of RV constraints from the models of the systems, especially those relating state variables across pairs of comparison. If the qualitative models and behaviors are extended to include quantitative information, a form of semi-quantitative comparative analysis could be considered as well.

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