

Liu XuHua

Dept. of Comp. Scie.
Jilin University
Changchun, PRC.

ABSTRACT

The author proposed concepts of Operator Fuzzy Logic and λ -Resolution in 1984. He and his cooperaters have obtained some theoretical results. This paper introduces λ -paramodulation to handle a set of clauses with the predicate of equality, and thus the equality substitution can be used in fuzzy reasoning. Then the λ -paramodulation method is proved to be complete with λ -lock-semantic method. Finally, Yang Fengjie and I have improved the λ -paramodulation, and proved that the linear λ -paramodulation is complete too.

I. INTRODUCTION

The author proposed concepts of Operator Fuzzy Logic(or OFL for short) and λ -Resolution in 1984. During the past few years (1985-1987), he and H. Xiao, then he, K.Y.Fang, Carl.K.Chang and Jeff. J-P Tsai working together on the λ -resolution method obtained a series of results. We proved that a λ -resolvent of two fuzzy clauses C_1 and C_2 is a λ -logical consequence of C_1 and C_2 , and that λ -resolution method is complete for λ -inconsistent set of clauses. We explained the practical meaning of fuzzy reasoning based on λ -resolution method in OFL. From the above results, we can see that some theorems which can not be proved with the traditional logic, can be proved fuzzily by using λ -resolution method in our logic system.

Equality relation is a very important relation in mathematics. This equality relation has some important special properties: it is transitive and can substitute equals for equals. Thus it is natural that the transitive and substitutive properties of fuzzily equality are needed in proving fuzzy theorems.

This paper proposed a λ -paramodulation to handle fuzzy equality. In conjunction with λ -resolution, λ -paramodulation can be used to prove fuzzy theorems in OFL. We proved that λ -paramodulation is complete for the λ -inconsistent set of clauses in conjunction with λ -resolution.

II. FUNDAMENTAL CONCEPTS AND PROPERTIES

From [1-3], we know that Operator Fuzzy Logic is built on operator lattice and interval $[0,1]$ is an operator lattice if we define:

$$\begin{aligned}x * y &= \min\{x, y\}, \\x \oplus y &= \max\{x, y\}, \\x \cdot y &= (x + y) / 2, \\x' &= 1 - x,\end{aligned}$$

for any $x, y \in [0,1]$.

In the following discussion, we assume that OFL is built on interval $[0,1]$ and the amount of operator λ is finite.

These concepts such as operator lattice, fuzzy literal, formula, interpretation, truth-value $T_I(G)$ of formula G under I , λ -complemented literal, λ -identical literal, can be found in Refs. [1-3].

Definition 1. Choose arbitrarily $\lambda \in [0,1]$. Formula G is called λ -valid if and only if for every I there exists $T_I(G) \geq \lambda$; G is called λ -inconsistent if and only if for every I , there exists $T_I(G) < \lambda$.

Obviously, Formula G is λ -valid if and only if Formula $(\sim G)$ is $(1-\lambda)$ -inconsistent.

Definition 2. Let C_1 and C_2 be two clauses without the same variables and $\lambda_1 L_1$ and $\lambda_2 L_2$ be two literals of C_1 and C_2 respectively. If L_1 and L_2 have a Most General Unifier (MGU for short) σ , and $\lambda_1 L_1$ and $\lambda_2 L_2$ are λ -complemental, then $(C_1 - S_1) \cup (C_2 - S_2)$ is the binary λ -resolvent for C_1 and C_2 , denoted by $R_\lambda(C_1, C_2)$, where

$$S_1 = \{ \lambda * L^{\epsilon} : (\lambda * L^{\epsilon} \in C_1) \wedge (\lambda * L^{\epsilon} \text{ and } \lambda_1 L_1^{\epsilon} \text{ are } \lambda\text{-identical}) \},$$

$$S_2 = \{ \lambda * L^{\epsilon} : (\lambda * L^{\epsilon} \in C_2) \wedge (\lambda * L^{\epsilon} \text{ and } \lambda_2 L_2^{\epsilon} \text{ are } \lambda\text{-identical}) \}.$$

The following properties are simple, and so their proofs are omitted.

Property 1. Let P be an atom, then $\neg(1P)=0P$.

Property 2. Let P be an atom, then $\neg(0P)=1P$.

Henceforth, 1P can be denoted by P in OFL.

Property 3. Let P be an atom, then $\neg(\lambda P)=(1-\lambda)P$.

Property 4. Let P be an atom, then $\neg(\lambda_1 \dots \lambda_n P)=(1-\lambda_1) \dots (1-\lambda_n)P$.

Property 5. Let G be formula, then $\neg G \approx 0G$.

For example, Let $G=0.3P$, $I=\{P\}$, then $T_I(\neg G)=T_I(0.7P)=0.7$.

$T_I(0G)=T_I(0(0.3P))=0.15$.

Property 6. Let G be a formula, then $\neg \lambda G=(1-\lambda)(\neg G)$.

Property 7. Let G be a formula, then $\neg(\lambda_1 \dots \lambda_n G)=(1-\lambda_1) \dots (1-\lambda_n)(\neg G)$.

Property 8. Let P be an atom, then $\lambda_1 \lambda_2 P \approx (\lambda_1 * \lambda_2)P$.

For example, $0.8(0.6P) \approx (0.8 * 0.6)P=0.7P$.

Property 9. Let G be a formula, then $\lambda_1 \lambda_2 G \approx (\lambda_1 * \lambda_2)G$.

Definition 3. Let G and H be two formulas, I be any interpretation. If $T_I(H) \geq \lambda$ for $T_I(G) > \lambda$, then it is said that G λ -implies H, denoted by $G \Rightarrow H$.

Definition 4. Let G and H be two formulas and I be any interpretation.

If $T_I(H) > \lambda$ for $T_I(G) > \lambda$, then it is said that G λ -strong implies H, denoted by $G \Rightarrow H$.

Property 10. Let A be a formula, then

- 1) $A \Rightarrow A$ for $\lambda \leq 0.5$.
- 2) $A \Rightarrow A$.

Property 11. Let A, B and C be formulas. Then

- 1) if $A \Rightarrow B, B \Rightarrow C$ then $A \Rightarrow C$ for $\lambda > 0.5$;
- 2) if $A \Rightarrow B, B \Rightarrow C$ then $A \Rightarrow C$.

Property 12. Let C_1 and C_2 be two clauses and $R_{\lambda}(C_1, C_2)$ be a λ -resolvent of C_1 and C_2 . Then

- 1) $C_1 \wedge C_2 \Rightarrow R_{\lambda}(C_1, C_2)$ for $\lambda=0.5$;
- 2) $C_1 \wedge C_2 \Rightarrow R(C_1, C_2)$ for $\lambda \geq 0.5$.

The proof of this property will be given by another paper (to appear).

Property 13. Let A, B and C be three formulas. Then

- 1) if $A \Rightarrow B, A \Rightarrow C$, then $A \Rightarrow (B \wedge C)$;
- 2) if $A \Rightarrow B, A \Rightarrow C$, then $A \Rightarrow (B \wedge C)$.

Definition 5. A clause C is called a λ -empty clause (denoted by $\lambda\text{-}\square$), if for any literal $\lambda * L^{\epsilon} \in C$, it satisfies the condition:

$$1-\lambda \leq \lambda * \lambda \leq \lambda$$

for $\lambda \geq 0.5$.

Property 14. Let $\lambda \geq 0.5$. A set of clauses is λ -inconsistent if and only if there is a resolution deduction of the λ -empty clause from S.

This property is an important theorem in Ref. [1] and another paper is

waiting for publishing. Its proof is omitted here.

III. THE SET OF FUZZY EQUALITY AXIOMS

Definition 6. An E-interpretation I_E of set S of clauses is an interpretation of S satisfying the four following conditions. Let α, β and γ be any terms in the Herbrand universe^[4] of S, and let $\lambda P(\dots \alpha \dots)$ be any fuzzy literal in S. Then

1. $T_{I_E}((\alpha=\alpha))=T$;
2. if $T_{I_E}((\alpha=\beta))=T$, then $T_{I_E}((\beta=\alpha))=T$;
3. if $T_{I_E}((\alpha=\beta))=T$, if $T_{I_E}((\beta=\gamma))=T$, then $T_{I_E}((\alpha=\gamma))=T$;
4. if $T_{I_E}((\alpha=\beta))=T$, then $T_{I_E}(P(\dots \alpha \dots))=T_{I_E}(P(\dots \beta \dots))$.

Definition 7. Let S be a set of clauses, $\lambda \in [0,1]$ and $\lambda \geq 0.5$. Then the set K_{λ} of λ -equality axioms for S is the set consisting of the following clauses: for any $\lambda^* > \lambda$,

1. $\lambda^*(x=x)$;
2. $(1-\lambda^*)(x=y) \vee \lambda^*(y=x)$;
3. $(1-\lambda^*)(x=y) \vee (1-\lambda^*)(y=z) \vee \lambda^*(x=z)$;
4. $(1-\lambda^*)(x_j=x_0) \vee (1-\lambda^*)P(\dots x_j \dots) \vee \lambda^*P(\dots x_0 \dots)$,

for any atom $P(x_1, \dots, x_n)$ occurring in S, $j=1, \dots, n$;

5. $(1-\lambda^*)(x_j=x_0)$

$\vee \lambda^*(f(\dots x_j \dots)=f(\dots x_0 \dots))$, for any function symbol $f(x_1, \dots, x_n)$ occurring in S, $j=1, \dots, n$.

Definition 8. A set S of clauses is called λE -inconsistent if and only if $T_{I_E}(S) \leq \lambda$ for any E-interpretation I_E ; S is called λE -valid if and only if $T_{I_E}(S) \geq \lambda$, for any I_E .

If I_E is an E-interpretation of S, we can obtain results obviously as follows:

1. $T_{I_E}(\lambda^*(x=y))=\lambda^* > \lambda$.
2. $T_{I_E}((1-\lambda^*)(x=y) \vee \lambda^*(y=x))=\lambda^* > \lambda$.
3. $T_{I_E}((1-\lambda^*)(x=y) \vee (1-\lambda^*)(y=z) \vee \lambda^*(x=z))=\lambda^* > \lambda$.

4. If $T_{I_E}((x_j=x_0))=T$, because I_E is an E-interpretation, then $T_{I_E}(P(\dots x_j \dots))=T_{I_E}(P(\dots x_0 \dots))$, and therefore,

$$T_{I_E}((1-\lambda^*)(x_j=x_0) \vee (1-\lambda^*)P(\dots x_j \dots) \vee \lambda^*P(\dots x_0 \dots))=\lambda^* > \lambda.$$

5. If $T_{I_E}((x_j=x_0))=T$, because I_E is an E-interpretation, then

$$T_{I_E}(f(\dots x_j \dots))=T_{I_E}(f(\dots x_0 \dots))=T_{I_E}(f(\dots x_j \dots)=f(\dots x_0 \dots)),$$

and since $T_{I_E}((\alpha=\alpha))=T$ for any α , we have $T_{I_E}(f(\dots x_j \dots)=f(\dots x_0 \dots))=T$.

Thus

$$T_{I_E}((1-\lambda^*)(x_j=x_0) \vee \lambda^*(f(\dots x_j \dots)=f(\dots x_0 \dots)))=\lambda^* > \lambda.$$

From the above discussion, we can easily see that $T_{I_E}(K_{\lambda}) > \lambda$, where K_{λ} is a set of λ -equality axioms of S.

If I is an interpretation of S and $T_I(K_\lambda) > \lambda$, we can obtain results obviously as follows:

1. $T_I((\alpha = \alpha)) = T$ for any term α in S .
2. If $T_I((\alpha = \beta)) = T$, because $T_I(K_\lambda) > \lambda$, thus

$T_I((1 - \lambda^*)(\alpha = \beta) \vee \lambda^*(\beta = \alpha)) > \lambda$, therefore, there must be

$$T_I(\beta = \alpha) = T.$$

3. Let $T_I((\alpha = \beta)) = T$, $T_I((\beta = \gamma)) = T$. According to $T_I(K_\lambda) > \lambda$, thus $T_I((1 - \lambda^*)(\alpha = \beta) \vee (1 - \lambda^*)(\beta = \gamma) \vee \lambda^*(\alpha = \gamma)) > \lambda$, there must be $T_I((\alpha = \gamma)) = T$

4. Suppose $T_I((\alpha = \beta)) = T$,

$$T_I(P(\dots \alpha \dots)) = T. \text{ Since}$$

$$T_I((1 - \lambda^*)(\alpha = \beta) \vee (1 - \lambda^*)P(\dots \alpha \dots) \vee \lambda^*P(\dots \beta \dots)) > \lambda,$$

there must be $T_I(P(\dots \beta \dots)) = T$.

Similarly, we can easily see that if

$$T_I((\alpha = \beta)) = T, T_I(P(\dots \beta \dots)) = T,$$

then there must be

$$T_I(P(\dots \alpha \dots)) = T, \text{ namely,}$$

$$T_I(P(\dots \alpha \dots)) = T_I(P(\dots \beta \dots)).$$

From the above discussion, we can easily see that I is an E-interpretation of S .

Therefore, we can obtain a theorem as follows:

Theorem 1. Let S be a set of clauses and K_λ be the set of λ -equality axioms for S . For any interpretation I of S , I is an E-interpretation if and only if

$$T_I(K_\lambda) > \lambda.$$

Theorem 2. Let S be a set of clauses and K_λ be the set of λ -equality axioms for S . Then S is λ E-inconsistent if and only if (SUK_λ) is λ -inconsistent.

($\Rightarrow$) Suppose S is λ E-inconsistent, but (SUK_λ) is not λ -inconsistent. Then there exists an interpretation I such that $T_I(SUK_\lambda) > \lambda$.

Thus $T_I(S) > \lambda$ and $T_I(K_\lambda) > \lambda$.

From theorem 1, we know that I must be an E-interpretation. From $T_I(S) > \lambda$, we know that S is not λ E-inconsistent, which contradicts the assumption that S is λ E-inconsistent.

($\Leftarrow$) Suppose (SUK_λ) is λ -inconsistent, but S is not λ E-inconsistent. Then there exists an E-interpretation I_E such that $T_{I_E}(S) > \lambda$. Clearly, $T_{I_E}(K_\lambda) > \lambda$. Hence $T_{I_E}((SUK_\lambda)) > \lambda$. This contradicts the assumption that (SUK_λ) is λ -inconsistent.

IV. λ -PARAMODULATION

Definition 9. Let $\lambda \geq 0.5$ and C_1, C_2 be two clauses without any variables in common such that

$$\lambda_1 L[t]VC_1', \lambda_1 > \lambda \text{ or } \lambda_1 < 1 - \lambda, \\ \lambda_2(r=s)VC_2', \lambda_2 > \lambda,$$

where $\lambda_1 L[t]$ is a fuzzy literal containing the term t and C_1' and C_2' are clauses. If t and r have MGU σ , then

$$\lambda^* L^\sigma[s^\sigma]UC_1'^\sigma UC_2'^\sigma$$

is called a binary λ -paramodulant of C_1 and C_2 , $\lambda_1 L[t]$ and $\lambda_2(r=s)$ are called λ -paramodulated literals where $L^\sigma[s^\sigma]$ denotes the result obtained by replacing one single occurrence of t^σ in L^σ by s^σ , and

$$\lambda^* = \begin{cases} (\lambda_1 + \lambda_2)/2 & \text{when } \lambda_1 > \lambda, \\ (\lambda_1 + 1 - \lambda_2)/2 & \text{when } \lambda_1 < 1 - \lambda. \end{cases}$$

Definition 10. A λ -paramodulant of C_1 and C_2 is a binary λ -paramodulant of C_1 or a λ -factor of C_1 and C_2 or a λ -factor of C_2 , denoted by $P_\lambda(C_1, C_2)$.

Definition 11. Let S be a set of clauses; G and H be two clauses in S . If $T_{I_E}(H) > \lambda$ when $T_{I_E}(G) > \lambda$ for every E-interpretation I_E of S , then it is said that G λ E-strongly implies H , denoted by $G \Rightarrow_\lambda H$.

Theorem 3. Let C_1 and C_2 be two clauses, $\lambda \geq 0.5$, and suppose $P_\lambda(C_1, C_2)$ is a λ -paramodulant. Then

$$(C_1 \wedge C_2) \Rightarrow_\lambda P_\lambda(C_1, C_2).$$

Without loss of generality, we may assume that C_1 is $\lambda_1 L[t]VC_1'$, C_2 is $\lambda_2(r=s)VC_2'$, $P_\lambda(C_1, C_2)$ is a binary λ -paramodulant, with $\lambda_2 > \lambda$. Let MGU of t and r be σ , I_E be any one of E-interpretation such that

$$T_{I_E}(C_1) > \lambda \text{ and } T_{I_E}(C_2) > \lambda.$$

Then, Obviously, we have

$$T_{I_E}(C_1^\sigma) > \lambda \text{ and } T_{I_E}(C_2^\sigma) > \lambda.$$

(Notice: every variable in C_1 and C_2 is considered to be governed by a universal quantifier). Choosing an arbitrarily ground instance of C_1 and a ground instance of C_2 , we have the following considerations:

1. For $\lambda_1 > \lambda$

$$\text{If } T_{I_E}(L^\sigma[t^\sigma]) = F, \text{ Then } T_{I_E}(C_1^\sigma) > \lambda$$

, thus $T_{I_E}(P_\lambda(C_1, C_2)) > \lambda$.

If $T_{I_E}(L^\sigma[t^\sigma]) = T$, we have $T_{I_E}(L^\sigma[s^\sigma]) = T$; because $t^\sigma = r^\sigma$ and $r^\sigma = s^\sigma$, we know $\lambda^* > \lambda$ (λ^* occurring in the definition of $P_\lambda(C_1, C_2)$), therefore $T_{I_E}(P_\lambda(C_1, C_2)) > \lambda$.

2. For $\lambda_1 < 1 - \lambda$

similarly to the above proof, we can also obtain

$$T_{I_E}(P_\lambda(C_1, C_2)) > \lambda.$$

According to the above proof, we have the following fact:

$$(C_1 \wedge C_2) \Rightarrow_\lambda P_\lambda(C_1, C_2).$$

Q.E.D.

Definition 12. Let C_1 and C_2 be two clauses, where every literal is locked with an integer and $\lambda \geq 0.5$ and let $P_\lambda(C_1, C_2)$ be a λ -paramodulant. Suppose

1. C_1 and C_2 are λ -positive clause, namely, every fuzzy literal $\lambda^* L$ in clauses satisfies $\lambda^* \geq 1 - \lambda$.

2. the paramodulated literal of C_i contains the smallest lock in C_i ($i=1,2$). Then $P_\lambda(C_1, C_2)$ is called a λ -lock-hyperparamodulant, or a λ -LH-paramodulant for short.

Definition 13. Let $\lambda \geq 0.5$. A finite set of clauses $\{E_1, \dots, E_q, N\}$ where every literal is locked with an integer is called a λ -lock-hypersemantic clash (or λ -LH-Clash for short) if and only if

$E_1, \dots, E_q$ satisfies the following conditions:

1. $E_1, \dots, E_q$ are λ -positive clauses.
2. Let $R_i = N$. For each $i (1 \leq i \leq q)$, there is a λ -resolvent R_{i+1} of E_i and R_i .
3. The resolved literal of E_i contains the smallest lock in E_i .
4. R_{q+1} is a λ -positive clause, where R_{q+1} is called a λ -LH-resolvent.

Theorem 4. Let S be a set of clauses, where every literal is locked with an integer and $\lambda \geq 0.5$. If S is λ -inconsistent, then there is a deduction of λ -empty clause from S by the λ -LH-resolution method.

proof. see [3].

Definition 14. Let S be a set of clauses and $\lambda \geq 0.5$. The set F_λ of λ -functionally reflexive axioms for S is the set defined as

$F_\lambda = \{\lambda^*(f(x_1, \dots, x_n) = f(x_1, \dots, x_n))\}$, where $\lambda^* \in [0, 1]$ and $\lambda^* > \lambda$ and f is any function symbol occurring in S .

Definition 15. Let $\lambda \geq 0.5$. The fuzzy literal λ^*L is called a λ -irrelevant literal if $1 - \lambda \leq \lambda^* \leq \lambda$.

Definition 16. Let C_1 and C_2 be two clauses. If we can build the one-to-one correspondence between the literals of C_1 and of C_2 such that the two corresponding literals are either λ -identical or λ -irrelevant, then C_1 is called λ -identical with C_2 .

Theorem 5. Let $\lambda \geq 0.5$, C_1 and C_2 be two clauses, C be a λ -lock-resolvent or a λ -lock-paramodulant of C_1 and C_2 . If C_1^* and C_2^* are λ -identical with C_1 and C_2 respectively, then there is a λ -lock-resolvent C^* or a λ -lock-paramodulant C^* such that C^* is λ -identical with C .

Proof 1. Let C_1 be $\lambda_1 L_1 VC_1'$ and C_2 be $\lambda_2 L_2 VC_2'$, where $\lambda_1 > \lambda$ and $\lambda_2 < 1 - \lambda$. Let $C = (C_1' - \lambda_1 L_1') \cup (C_2' - \lambda_2 L_2')$, where σ is an MGU of L_1 and L_2 .

It is clear that C_1^* and C_2^* are two clauses as follows:

$\lambda_1^* L_1 VC_1^{*'} , \lambda_2^* L_2 VC_2^{*'} ,$
where $\lambda_1^* > \lambda$, $\lambda_2^* < 1 - \lambda$, $C_1^{*'}$ and $C_2^{*'}$ are λ -identical with C_1' and C_2' respectively. Let

$C^* = (C_1^{*'} - \lambda_1^* L_1^{*'}) \cup (C_2^{*'} - \lambda_2^* L_2^{*'})$. Obviously, C^* is a λ -resolvent of C_1^* and C_2^* and is also a λ -lock-resolvent, C^* being λ -identical with C .

2. We might as well assume that C_1 is $\lambda_1 L[t]VC_1'$ and C_2 is $\lambda_2 (r=s)VC_2'$, where $\lambda_1 > \lambda$ and $\lambda_2 > \lambda$. Let

$C = \lambda_3 L^\sigma [s^\sigma] VC_1'^\sigma VC_2'^\sigma$,
where $\lambda_3 = (\lambda_1 + \lambda_2)/2$ and σ is an MGU of t and r .

Then, C_1^* and C_2^* are two clauses as follows:

$\lambda_1^* L[t]VC_1^{*'} ,$
 $\lambda_2^* (r=s)VC_2^{*'} ,$
where $\lambda_1^* > \lambda$, $\lambda_2^* > \lambda$, $C_1^{*'}$ and $C_2^{*'}$ are λ -identical with C_1' and C_2' respectively. Let

$$C^* = \lambda_3^* L^\sigma [s^\sigma] VC_1^{*'} VC_2^{*'} ,$$

where $\lambda_3^* = (\lambda_1^* + \lambda_2^*)/2$. Obviously, C^* is a λ -lock-paramodulant and is λ -identical with C . Q.E.D.

Theorem 6. Let $\lambda \geq 0.5$ and S be a set of clauses, where every literal is locked with an integer. Then, S is λE -inconsistent if and only if there is a deduction of λ -empty clause from $(SU\{\lambda^*(x=x)\}UF_\lambda)$ by λ -LH-resolution method and λ -LH-paramodulation method, where $\lambda^* > \lambda$ and F_λ is the set of λ -functionally reflexive axioms for S . (λ -empty clause is composed of irrelevant literals, denoted by $\lambda-\square$.)

($\Rightarrow$). Suppose K_λ is the set of λ -functionally reflexive axioms for S . Because S is λE -inconsistent, we know from theorem 2 that (SUK_λ) is λ -inconsistent. From theorem 4, we know that there is a λ -LH-resolution deduction D of λ -empty clause from (SUK_λ) . We now show that D can be transformed into the deduction satisfying this theorem.

For each λ -LH-clash $(E_1, \dots, E_q, N)$ in D , since $E_1, \dots, E_q$ are λ -positive clauses, $E_1, \dots, E_q$ must be in S or $\lambda^*(x=x)$. If $N \notin S$, then the clash is the desired one. If $N \in K_\lambda$, N is clearly not $\lambda^*(x=x)$. Then we have the following four possible cases:

1. N is $(1 - \lambda^*)(x=y) \vee \lambda^*(y=x)$ and $\lambda^* > \lambda$. Obviously, there must be $q=1$ and $E_1 = (\lambda_1(t_1=t_2) \vee E_1')$, where $\lambda_1 > \lambda$. Therefore,

$$(E_1, N) = (\lambda^*(t_1=t_2) \vee E_1') .$$

In addition,

$$P_\lambda(\lambda_1(x=x), \lambda_1(t_1=t_2) \vee E_1') = (\lambda_1(t_2=t_1) \vee E_1') .$$

Because (E_1, N) is a λ -LH-clash, $P_\lambda(\lambda_1(x=x), E_1)$ must be a λ -LH-paramodulant and (E_1, N) is λ -identical with $P_\lambda(\lambda_1(x=x), E_1)$.

2. N is $(1 - \lambda^*)(x=y) \vee (1 - \lambda^*)(y=z) \vee \lambda^*(x=z)$ and $\lambda^* > \lambda$. Obviously, we have $q=2$ and two clauses E_1 and E_2 as follows:

$\lambda_1^*(t_1=t_2) \vee E_1' , \lambda_2^*(s_1=s_2) \vee E_2' ,$
where $\lambda_1^* > \lambda$, $\lambda_2^* > \lambda$. Thus,

$(E_1, E_2, N) = (\lambda^*(t_1^\sigma = s_2^\sigma) \vee E_1'^\sigma \vee E_2'^\sigma)$,
 σ being an MGU of t_2 and s_1 . Clearly,
 $P_\lambda(E_1, E_2) = (\lambda_1^* + \lambda_2^*)/2 (t_1^\sigma = s_2^\sigma) \vee E_1'^\sigma \vee E_2'^\sigma$. We can see that $P_\lambda(E_1, E_2)$ is a λ -LH-paramodulant and λ -identical with (E_1, E_2, N) .

3. N is $(1 - \lambda^*)(x_j = x_0) \vee (1 - \lambda^*)P(\dots x_j \dots) \vee \lambda^*P(\dots x_0 \dots)$, with $\lambda^* > \lambda$. Obviously, we have $q=2$ and E_1 and E_2 are two clauses as follows:

$\lambda_1(t_j=t_0) \vee E_1' , \lambda_2 P(\dots s_j \dots) \vee E_2' ,$
where $\lambda_1 > \lambda$ and $\lambda_2 > \lambda$. Thus
 $(E_1, E_2, N) = (\lambda^* P(s_1^\sigma \dots t_0^\sigma \dots s_n^\sigma) \vee E_1'^\sigma \vee E_2'^\sigma)$,
where σ is an MGU of $(x_1 \dots t_j \dots x_n)$ and $(s_1 \dots s_j \dots s_n)$.

Clearly,
 $P_\lambda(E_2, E_1) = (\lambda_1 + \lambda_2)/2 P(s_1^\sigma \dots t_0^\sigma \dots s_n^\sigma) \vee E_1'^\sigma \vee E_2'^\sigma$.

We can see that $P_{\lambda}(E_2, E_1)$ is a λ -LH-paramodulation and λ -identical with (E_1, E_2, N) .

4. N is $(1-\lambda^*)(x_j=x_0)$

$$\forall \lambda^*(f(\dots x_j \dots)=f(\dots x_0 \dots)),$$

where $\lambda^* > \lambda$.

Obviously, we have $q=1$ and E_1 is a clause as follows:

$$\lambda_1(t_j=t_0) \vee E_1',$$

where $\lambda_1 > \lambda$. Thus,

$$(E_1, N)=$$

$(\lambda^*(f(x_1 \dots t_j \dots x_n)=f(x_1 \dots t_0 \dots x_n))) \vee E_1')$
Clearly, λ -paramodulant of

$\lambda_1(f(x_1 \dots x_j \dots x_n)=f(x_1 \dots x_j \dots x_n))$ and E_1 is a clause as follows:

$$P_{\lambda}(\lambda_1(f=f), E_1)=$$

$(\lambda_1(f(x_1 \dots t_j \dots x_n)=f(x_1 \dots t_0 \dots x_n))) \vee E_1')$
We can see that $P_{\lambda}(\lambda_1(f=f), E_1)$ is a λ -LH-paramodulant and λ -identical with (E_1, N) .

From Theorem 5, we know that each λ -LH-Clash in D can be transformed into a λ -LH-paramodulant of two clauses which belong to the set $(SU\{\lambda^*(x=x)\} \cup F_{\lambda})$.

Therefore, we obtain a new deduction D' of λ -empty from $(SU\{\lambda^*(x=x)\} \cup F_{\lambda})$ by using λ -LH-resolution method and λ -LH-paramodulation method.

($\Leftarrow$). If there is a deduction of λ -empty clause from $(SU\{\lambda^*(x=x)\} \cup F_{\lambda})$ by using λ -LH-resolution and λ -LH-paramodulation, but S is not λE -inconsistent, then there is an E -interpretation I_E such that $T_{I_E}(S) > \lambda$. Thus, for each clause C in $(SU\{\lambda^*(x=x)\} \cup F_{\lambda})$, there must be $T_{I_E}(C) > \lambda$. From Property 11, Property 12 and theorem 3, we can easily see that $T_{I_E}(\lambda-\square) > \lambda$, which contradicts the definition of $\lambda-\square$.

Q.E.D.

V. LINEAR λ -PARAMODULATION

Definition 17. Given a set S of clauses and a clause C_0 in S , a linear λ -deduction of C_n from S with top clause C_0 by λ -resolution and λ -paramodulation is a deduction of the form shown in Fig. 1, where

1. For $i=1, \dots, n-1$, C_{i+1} is a λ -resolvent or a λ -paramodulation of C_i (called a center clause) and B_i (called a side clause).

2. Each B_i is either in S , or is a C_j for some $j < i$.

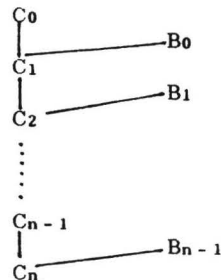


Figure 1

A Linear λ -refutation
by λ -resolution and λ -paramodulation is a linear λ -deduction of $\lambda-\square$ by λ -resolution and λ -paramodulation.

Definition 18. Let $\lambda \geq 0.5$, and S be a set of clauses. S_R is said to be a λ -reduction of S if and only if S_R is obtained by the following replacing: for any literal $\lambda^* L$ S ,

1. if $\lambda^* \leq \lambda$ and $1-\lambda \leq \lambda^*$, delete $\lambda^* L$.

2. if $\lambda^* > \lambda$, $\lambda^* L$ is replaced by L .

3. if $1-\lambda > \lambda^*$, $\lambda^* L$ is replaced by $\sim L$.

Let S_R denote the λ -deduction of S , C_R denote the λ -deduction of C .

Theorem 7. Let C_{1R} and C_{2R} be the λ -reducing clauses of C_1 and C_2 respectively. If C' is a resolvent (or a paramodulant) of C_{1R} and C_{2R} , then there is a λ -resolvent (or a λ -paramodulant) C of C_1 and C_2 such that $C_R = C'$.

Proof. If C' is a resolvent of C_{1R} and C_{2R} , we know that this theorem is correct. [1]

If C' is a paramodulation of C_{1R} and C_{2R} , without loss of generality, we may assume that C' is a binary paramodulant, let

$$C_1 = \lambda_1 L[t] \vee C_1', \quad \lambda_1 > \lambda \text{ or } \lambda_1 < 1-\lambda,$$

$$C_2 = \lambda_2 (r=s) \vee C_2', \quad \lambda_2 > \lambda.$$

Then

$$C_{1R} = \begin{cases} L[t] \vee C_{1R}', & \lambda_1 > \lambda, \\ \sim L[t] \vee C_{1R}', & \lambda_1 < 1-\lambda, \end{cases}$$

$$C_{2R} = (r=s) \vee C_{2R}'.$$

Therefore $C' = \begin{cases} L^\sigma[s^\sigma] \vee C_{1R}'^\sigma \vee C_{2R}'^\sigma, & \lambda_1 > \lambda, \\ \sim L^\sigma[s^\sigma] \vee C_{1R}'^\sigma \vee C_{2R}'^\sigma, & \lambda_1 < 1-\lambda. \end{cases}$

Since

$$C = \begin{cases} [(\lambda_1 + \lambda_2)/2] L^\sigma[s^\sigma] \vee C_{1R}'^\sigma \vee C_{2R}'^\sigma, & \lambda_1 > \lambda, \\ [(\lambda_1 + 1 - \lambda_2)/2] L^\sigma[s^\sigma] \vee C_{1R}'^\sigma \vee C_{2R}'^\sigma, & \lambda_1 < 1-\lambda. \end{cases}$$

But when $\lambda_1 > \lambda$, $(\lambda_1 + \lambda_2)/2 > \lambda$

and when $\lambda_1 < 1-\lambda$, $(\lambda_1 + 1 - \lambda_2)/2 < 1-\lambda$.

Then

$$C_R = \begin{cases} L^\sigma[s^\sigma] \vee C_{1R}'^\sigma \vee C_{2R}'^\sigma, & \lambda_1 > \lambda, \\ \sim L^\sigma[s^\sigma] \vee C_{1R}'^\sigma \vee C_{2R}'^\sigma, & \lambda_1 < 1-\lambda. \end{cases}$$

$$= C'. \quad \text{Q.E.D.}$$

Theorem 8. If C is a clause in an λE -inconsistent set S of clauses including $\lambda^*(x=x)$ and the set F_{λ} of λ -functionally reflexive axioms for S and if $S - \{C\}$ is λE -satisfiable, then S has a linear λ -refutation by λ -resolution and λ -paramodulation with top clause C .

Proof. According the theorem above we know that S is λE -inconsistent if and only if $(S \cup K_{\lambda})$ is λ -inconsistent if and only if $S_R \cup K$ is inconsistent. (K is the λ -reduction of K_{λ}).

Since $S - \{C\}$ is λE -satisfiable, clearly, $S_R - \{C_R\}$ is E -satisfiable, then S_R has a linear refutation D by resolution and paramodulation with top clause C_R [4]. By theorem 7, for every resolvent $R(C_{1R}, C_{2R})$ or paramodulant $P(C_{1R}, C_{2R})$ in D , there is a λ -resolvent $R(C_1, C_2)$ or λ -paramodulation $P_\lambda(C_1, C_2)$, and

$$R(C_{1R}, C_{2R}) = (R_\lambda(C_1, C_2))_R$$

$$P(C_{1R}, C_{2R}) = (P_\lambda(C_1, C_2))_R.$$

Therefore, we can obtain a linear deduction of λ -empty clause from S , by λ -resolution and λ -paramodulation.

Up to now we have obtained λ -paramodulation method in operator fuzzy logic. This is an inference rule for the fuzzy equality relation. λ -paramodulation is essentially an extension of the fuzzy equality substitution.

Because OFL describes fuzzy propositions naturally [6], the combination of λ -resolution and λ -paramodulation is convenient in doing fuzzy reasoning.

References

- [1] Liu, X. H. & Xiao, H., Operator fuzzy logic and fuzzy resolution, Proc. of the 15th ISMVL, Canada, 5(1985), 68-75
- [2] Liu, X.H. & Fang, K.Y., Fuzzy reasoning on λ -horn set, Proc. of the 16th ISMVL, U.S.A., 5(1986), 248-251
- [3] Liu, X.H., Chang, Carl K. & Jeffery J-P Tsai, Fuzzy reasoning base on λ -LH-resolution, Proc. of the IEEE 10th International Computer Software Application Conference, U.S.A., 10(1986), 154-157
- [4] Chang, C. L. & Lee, R.C.T., Symbolic Logic and Mechanical Theorem Proving, Academic Press, New York, 1973.
- [5] Lee, R.T., Fuzzy Logic and the Resolution Principle, Journal of ACM, 19(1972), 1:109-119
- [6] Liu, X.H., Fang, K.Y. & Jeffery J-P Tsai, Fuzzy reasoning in operator fuzzy logic, Proc. of 19th ISMVL-89, China, 5, 1989.
- [7] Liu, X.H. & Yang, F.J. "Linear λ -paramodulation" (to appear).