

APPROXIMATE REASONING BASED ON FUZZY LOGIC

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During the past several years, the emergence of expert systems as a field of considerable practical as well as theoretical importance within AI has provided a strong impetus for the development of theories of approximate reasoning and credibility assessment of inference processes in knowledge-based systems.

The approach to approximate reasoning described in this paper is based on a fuzzy logic, FL, in which the truth-values and quantifiers are defined as possibility distributions which carry linguistic labels such as true, quite true, not very true, many, not very many, several, almost all, etc. Based on the concept of a possibility distribution, a set of translation and inference rules is developed and their application to inference from imprecise premises is illustrated by examples.

1. INTRODUCTION

It has long been recognized that much, perhaps most, of human reasoning is approximate rather than exact in nature.** And yet, logicians and cognitive scientists have devoted scant attention to the development of theories of approximate reasoning, partly because of the deeply entrenched tradition of respect for what is precise and disdain for what is fuzzy, and partly because there was no compelling need for such theories before the advent of artificial intelligence.

In recent years, however, the rapid growth of interest in natural language processing and the emergence of expert systems as an important application area within AI have made it increasingly clear that a better understanding of the

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**The terms "approximate reasoning" and "fuzzy reasoning" are frequently used interchangeably in the literature. A comprehensive exposition of the foundations of fuzzy reasoning may be found in the paper by B.R. Gaines [1].

processes of approximate reasoning is a prerequisite to the development of knowledge-based systems which can manipulate information that is imprecise, incomplete or not totally reliable.

In a series of papers starting in 1973 [2-7], we have advanced the view that conventional logical systems do not provide an appropriate basis for approximate reasoning and have suggested a fuzzy logic, FL, for this purpose. In contrast to two-valued and multi-valued logics, the truth-values of FL are linguistic rather than numerical, as are quantifiers exemplified by many, most, not very many, few, several, almost all, etc. Furthermore, the rules of inference in FL are approximate rather than exact. Thus, in spirit as well as in substance FL represents a sharp break with the long-standing tradition of extreme precision in logic and a move toward an accommodation with the pervasive imprecision of human thought, perception, language and decision-making.

Another important difference between FL and classical logical systems centers on the concept of truth. Thus, whereas in the latter systems [8,9] the truth of a proposition serves as a starting point for the definition

of various basic concepts, the corresponding role in FL is played not by truth but by the concept of a possibility distribution, by which is meant a mapping associated with a variable X , $\pi_X: U \rightarrow [0,1]$, which assigns to each point u in the domain of X , U , the possibility that X could take u as a value. The concept of truth, then, is employed to express the compatibility of the possibility distribution induced by a proposition p with that of a reference proposition r . In this sense, the truth of a proposition is a relative concept and is a measure of its compatibility or consistency with a collection of propositions which constitute a database.

Our main concern in the present paper is with (a) the establishment of translation rules for various types of imprecise propositions; (b) inference from such propositions; and (c) the development of a conceptual framework for dealing with the issues of belief and credibility. The latter issues play a particularly important role in expert systems [10-17] as well as in the theories of evidence and decision analysis [18-24].

2. THE CONCEPT OF A POSSIBILITY DISTRIBUTION

As a preliminary to the consideration of translation rules for various types of propositions, it will be helpful to establish some of the properties of possibility distributions which will be needed in later sections.*

Informally, if X is a variable taking values in U , then a possibility distribution, π_X , associated with X may be viewed as a fuzzy (or elastic) constraint on the values that may be assigned to X . Such a distribution is characterized by a possibility distribution function $\pi_X: U \rightarrow [0,1]$ which associates with each $u \in U$ the "degree of ease" or the possibility that X may take u as a value.

In some cases, the constraint on the values of X is physical in origin. For example, if X is the number of students in a classroom which has, say, 50 seats, then $\pi_X(u) = 1$ for u up to 50, with $\pi_X(u)$ gradually declining to 0 at, say, $u = 75$. In this case, an intermediate value of π_X , say $\pi_X(65) = 0.3$, would signify that, by some explicit or implicit criterion, the degree of ease with which 65 students could be crowded into the classroom is 0.3 (on the scale from 0 to 1).

*An introductory exposition of possibility theory may be found in [29]. A historical account of the concepts of possibility and probability is contained in [30].

In most cases, however, the possibility distribution which is associated with a variable is epistemic rather than physical in origin. A basic assumption in fuzzy logic is that such epistemic possibility distributions are induced by propositions expressed in a natural language. In more concrete terms, this assumption may be stated as the

Possibility Postulate. If F is a fuzzy subset of U characterized by its membership function $\mu_F: U \rightarrow [0,1]$, then the proposition "X is F" induces a possibility distribution π_X which is equal to F . Equivalently, "X is F" translates into the possibility assignment equation $\pi_X = F$, i.e.,

$$X \text{ is } F \rightarrow \pi_X = F \quad (1)$$

which signifies that the proposition "X is F" has the effect of constraining the values that may be assumed by X , with the possibility distribution π_X identified with F .

As a simple illustration of (1), if in the proposition "X is small" small is regarded as a label of a fuzzy subset of $U = \{0,1,2,\dots\}$ which is defined by*

$$\text{SMALL} = 1/0 + 1/1 + 0.8/2 + 0.6/3 + 0.4/4 + 0.2/5 \quad (2)$$

$$\begin{array}{ll} \text{then} & \text{Poss}\{X=0\} = 1 \\ & \text{Poss}\{X=1\} = 1 \\ & \text{Poss}\{X=2\} = 0.8 \\ & \text{Poss}\{X=5\} = 0.2 \end{array} \quad (3)$$

where $\text{Poss}\{X=u\} = \pi_X(u)$ is the possibility that X may take u as a value.

An important aspect of the concept of a possibility distribution is that it is nonstatistical in nature. As a consequence, if P_X is a probability distribution associated with X then the only connection between π_X and P_X is that impossibility (i.e., zero possibility) implies improbability but not vice-versa. Thus, π_X cannot be inferred from P_X nor can P_X be inferred from π_X .

As in the case of probabilities, one can define joint and conditional possibilities. Thus, if X and Y are variables taking values in U and V , respectively, then we can define the joint and conditional possibility distributions

*The notation $F = \mu_1/x_1 + \dots + \mu_n/x_n$ signifies that F is a fuzzy subset of the set $\{x_1, \dots, x_n\}$, with μ_i , $i=1, \dots, n$, being the grade of membership of x_i in F . Uppercase symbols (e.g., F , SMALL) are employed to denote names of sets (fuzzy or nonfuzzy).

through their respective distribution functions:

$$\pi_{(X,Y)}(u,v) = \text{Poss}\{X=u, Y=v\}, u \in U, v \in V \quad (4)$$

and

$$\pi_{(X|Y)}(u|v) = \text{Poss}\{X=u|Y=v\} \quad (5)$$

where (5) represents the conditional distribution function of X given Y.

If we know the distribution function of X and the conditional distribution function of Y given X, then we can construct the joint distribution function of X and Y by forming the conjunction ($\wedge \triangleq \min$)

$$\pi_{(X,Y)}(u,v) = \pi_X(u) \wedge \pi_{(Y|X)}(v|u) \quad (6)$$

However, unlike the identity that holds in the case of probabilities, we can also obtain $\pi_{(X,Y)}(u,v)$ by forming the conjunction of $\pi_{(X|Y)}(u|v)$ and $\pi_{(Y|X)}(v|u)$:

$$\pi_{(X,Y)}(u,v) = \pi_{(X|Y)}(u|v) \wedge \pi_{(Y|X)}(v|u) \quad (7)$$

In yet another deviation from parallelism with probabilities, the marginal possibility distribution function of X may be expressed in more than one way in terms of the joint and conditional possibility distribution functions. More specifically, we may have

$$(a) \quad \pi_X(u) = \vee_v \pi_{(X,Y)}(u,v) \quad (8)$$

where $\vee_v$ denotes the supremum over $v \in V$;

$$(b) \quad \pi_X(u) = \vee_v \pi_{(X|Y)}(u|v) \quad (9)$$

and

$$(c) \quad \pi_X(u) = \pi_{(X|Y)}(u, \tilde{v}(u)) \quad (10)$$

where, for a given u, $\tilde{v}(u)$ is the value of v at which $\pi_{(Y|X)}(v|u) = 1$, if $\tilde{v}(u)$ is defined for every $u \in U$.

Intuitively, (a) represents the possibility of assigning a value to X as perceived by an observer ((X,Y) observer) who observes the joint possibility distribution $\pi_{(X,Y)}$. Similarly,

(b) represents the perception of an observer ((X|Y) observer) who observes only the conditional possibility distribution $\pi_{(X|Y)}$ and is unconcerned with or unaware of $\pi_{(Y|X)}$. And (c) expresses the perception of an observer who assumes that v is assigned that value, if it exists, which makes $\pi_{(Y|X)}(v|u)$ equal to unity.

In relating π_X to $\pi_{(X|Y)}$ through the operator $\vee$ (supremum), we are tacitly invoking the principle of maximal restriction [4], which asserts that, in the absence of complete information about π_X , we should equate π_X to the maximal (i.e., least restrictive) possibility distribution which is consistent with the partial information about π_X . In the case of (a), for example, this would be the supremum of $\pi_{(X,Y)}(u,v)$ over $v \in V$.

As will be seen in Section 3, the concept of a conditional possibility distribution plays a basic role in the formulation of a generalized form of modus ponens and in defining a measure of belief. What is as yet an unsettled issue revolves around the question of how to derive $\pi_{(X|Y)}$ and $\pi_{(Y|X)}$ from $\pi_{(X,Y)}$. Somewhat different answers to this question are presented in [25-27]. It may well turn out to be the case that, in contrast to probabilities, there does not exist a unique solution to the problem and that, in general, the answer depends on the perspective of the observer.

3. POSSIBILITY THEORY AND FUZZY LOGIC

As was alluded to already, the concept of a possibility distribution and the possibility theory which is based on it, play a central role in fuzzy logic by providing a means for the representation of the meaning of imprecise premises and the generation of propositions which follow logically from them.

Typically, a variable in fuzzy logic is treated as a linguistic variable [4], that is, a variable whose values are represented as words or sentences in a natural or synthetic language, with each such value defining a possibility distribution in the domain of the variable. In effect, what this implies is that a linguistic variable is a microlanguage with its own syntax and semantics. For example, in the case of FL, the linguistic values of the variable Truth may be generated by a context-free grammar and interpreted by an attributed grammar [28]. Thus, starting with (a) the primary term true and its antonym false; and (b) a finite set of modifiers and connectives such as and, or, not, very, more or less, quite, extremely, etc., the linguistic value of Truth may be represented as

true	false
not true	not false
very true	very false
not very true	not very false
more or less true	more or less false
quite true	quite false
not quite true	not quite false

not true and not false
not very true and not very false

In FL, a linguistic truth-value is regarded as a composite label of a possibility distribution in the interval $[0,1]$, which is the set of truth-values in Lukasiewicz's L_{Aleph_1} logic. What this means is that in FL we generally deal not with numerically-valued truth-values but with their possibility distributions.

An important distinction between FL and L_{Aleph_1} is that in L_{Aleph_1} --as in all other multi-valued logics--there are only two quantifiers all and some, whereas in FL we can employ a large variety of fuzzy quantifiers exemplified by few, several, many, most, almost all, very many, not very many, etc. This feature of FL makes it possible to translate, and infer from, imprecise premises exemplified by

Most tall women are not very fat.
 Among the many men who are heavy smokers,
 quite a few are overweight and some are
 heavy drinkers.
 Carol has several close friends who have
 many children.

The meaning of a quantifier in FL is based on the concept of the cardinality of a fuzzy set. Thus, if F is a fuzzy subset of a finite set $U = \{u_1, \dots, u_n\}$ characterized as

$$F = \mu_1/u_1 + \dots + \mu_n/u_n$$

where μ_i is the grade of membership of u_i in F , $i=1, \dots, n$, then the power of F --which is roughly a measure of its cardinality--may be defined as

$$|F| = \sum_{i=1}^n \mu_i \quad (11)$$

Alternatively, and perhaps more appropriately, the cardinality of F may be defined as a fuzzy number, as is done in [6]. Thus, if the elements of F are sorted in descending order, so that $\mu_n \leq \mu_m$ if $n \geq m$, then the truth-value of the proposition

$$p \triangleq F \text{ has at least } n \text{ elements} \quad (12)$$

is defined to be equal to μ_n , while that of q ,

$$q \triangleq F \text{ has at most } n \text{ elements}, \quad (13)$$

is taken to be $1 - \mu_{n+1}$. From this it follows that the truth-value of the proposition r ,

$$r \triangleq F \text{ has exactly } n \text{ elements}, \quad (14)$$

is given by $\mu_n \wedge (1 - \mu_{n+1})$. An illustration of these alternative definitions of cardinality is provided by Example 5 in Section 4.

The translation rules for propositions expressed in a natural language form an important part of PRUF--a meaning representation language based on possibility theory [7].* In what follows, we shall present in a summarized form a subset of such rules which play a basic role in FL.

Translation Rules

1. Modifier rule. If

$$X \text{ is } F \rightarrow \Pi_X = F \quad (15)$$

$$\text{then } X \text{ is } mF \rightarrow \Pi_X = F^+$$

where m is a modifier such as not, very, more or less, etc., and F^+ is a modification of F induced by m . More specifically: If $m = \text{not}$, then $F^+ = F' = \text{complement of } F$, i.e.,

$$\mu_{F^+}(u) = 1 - \mu_F(u), \quad u \in U. \quad (16)$$

If $m = \text{very}$, then $F^+ = F^2$, i.e.,

$$\mu_{F^+}(u) = \mu_F^2(u), \quad u \in U. \quad (17)$$

If $m = \text{more or less}$, then $F^+ = \sqrt{F}$, i.e.,

$$\mu_{F^+}(u) = \sqrt{\mu_F(u)}, \quad u \in U. \quad (18)$$

As a simple illustration of (15), if SMALL is defined as in (2), then

$$X \text{ is very small} \rightarrow \Pi_X = F^2 \quad (19)$$

where

$$F^2 = 1/0 + 1/1 + 0.64/2 + 0.36/3 + 0.16/4 + 0.04/5. \quad (20)$$

It should be noted that (16)-(18) should be viewed as default rules which may be replaced by other translation rules in cases in which some alternative interpretations of the modifiers very and more or less are more appropriate.

2. Conjunctive, disjunctive and implicational rules. If

$$X \text{ is } F \rightarrow \Pi_X = F \text{ and } Y \text{ is } G \rightarrow \Pi_Y = G \quad (21)$$

where F and G are fuzzy subsets of U and V ,

*There are several implemented languages which have extensive facilities for the manipulation of fuzzy propositions and execution of fuzzy instructions. Prominent among these are FUZZY [23] and FSTDS [30].

respectively, then

$$(a) \quad X \text{ is } F \text{ and } Y \text{ is } G \rightarrow \Pi_{(X,Y)} = F \times G \quad (22)$$

where

$$\mu_{F \times G}(u,v) \triangleq \mu_F(u) \wedge \mu_G(v) \quad (\wedge = \min) \quad (23)$$

$$(b) \quad X \text{ is } F \text{ or } Y \text{ is } G \rightarrow \Pi_{(X,Y)} = \bar{F} \cup \bar{G} \quad (24)$$

where

$$\bar{F} \triangleq F \times V, \quad \bar{G} \triangleq U \times G \quad (25)$$

and

$$\mu_{\bar{F} \cup \bar{G}}(u,v) = \mu_F(u) \vee \mu_G(v). \quad (26)$$

$$(c) \quad \text{If } X \text{ is } F \text{ then } Y \text{ is } G \rightarrow \Pi_{(Y|X)} = \bar{F}' \oplus \bar{G} \quad (27)$$

where $\Pi_{(Y|X)}$ denotes the conditional possibility distribution of Y given X , and the bounded sum $\oplus$ is defined by

$$\mu_{\bar{F}' \oplus \bar{G}}(u,v) = 1 \wedge (1 - \mu_F(u) + \mu_G(v)). \quad (28)$$

As simple illustrations of (22), (24) and (27), if

$$F \triangleq \text{SMALL} = 1/1 + 0.6/2 + 0.1/3 \quad (29)$$

$$G \triangleq \text{LARGE} = 0.1/1 + 0.6/2 + 1/3 \quad (30)$$

then

$$X \text{ is small and } Y \text{ is large} \rightarrow \Pi_{(X,Y)} \quad (31)$$

$$= 0.1/(1,1) + 0.6/(1,2) + 1/(1,3) + 0.1/(2,1) + 0.6/(2,2) + 0.6/(2,3) + 0.1/(3,1) + 0.1/(3,2) + 0.1/(3,3)$$

and

$$\text{If } X \text{ is small then } Y \text{ is large} \rightarrow \Pi_{(Y|X)} \quad (33)$$

$$= 0.1/(1,1) + 0.6/(1,2) + 1/(1,3) + 0.5/(2,1) + 1/(2,2) + 1/(2,3) + 1/(3,1) + 1/(3,2) + 1/(3,3).$$

3. Quantification rule. If $U = \{u_1, \dots, u_N\}$, Q is a quantifier such as many, few, several, all, some, most, etc., and

$$X \text{ is } F \rightarrow \Pi_X = F \quad (34)$$

then the proposition " QX are F " (e.g., " $\text{many } X\text{'s are large}$ ") translates into

$$\Pi_{\text{Count}(F)} = Q \quad (35)$$

where $\text{Count}(F)$ denotes the number (or the proportion) of elements of U which are in F . By definition (11), if the fuzzy set F is expressed as

$$F = \mu_1/u_1 + \mu_2/u_2 + \dots + \mu_N/u_N \quad (36)$$

then

$$\text{Count}(F) = \sum_{i=1}^N \mu_i. \quad (37)$$

As a simple illustration of (35), if the quantifier several is defined as

$$\text{SEVERAL} \triangleq 0/1 + 0.4/2 + 0.6/3 + 1/4 + 1/5 + 1/6 + 0.6/7 + 0.2/8 \quad (38)$$

then

$$\begin{aligned} \text{Several } X\text{'s are large} &\rightarrow \Pi_N \sum_{i=1}^N \mu_{\text{LARGE}}(u_i) \\ &= 0/1 + 0.4/2 + 0.6/3 + 1/4 + 1/5 + 1/6 \\ &\quad + 0.6/7 + 0.2/8 \end{aligned} \quad (39)$$

where $\mu_{\text{LARGE}}(u_i)$ is the grade of membership of the i th value of X in the fuzzy set LARGE .

4. Truth qualification rule. Let τ be a fuzzy truth-value, e.g., very true, quite true, more or less true, etc. Such a truth-value may be regarded as a fuzzy subset of the unit interval which is characterized by a membership function $\mu_\tau: [0,1] \rightarrow [0,1]$.

A truth-qualified proposition, e.g., "It is τ that X is F ," is expressed as " X is F is τ ." As shown in [5], the translation rule for such propositions is given by

$$X \text{ is } F \text{ is } \tau \rightarrow \Pi_X = F^+ \quad (40)$$

$$\mu_{F^+}(u) = \mu_\tau(\mu_F(u)). \quad (41)$$

As an illustration, consider the truth-qualified proposition

Bob is young is very true

which by (40), (41) and (17) translates into

$$\Pi_{\text{Age}(\text{Bob})} = \mu_{\text{TRUE}}^2(\mu_{\text{YOUNG}}(u)). \quad (42)$$

Now, if we assume that

$$\mu_{\text{YOUNG}}(u) = (1 + (\frac{u}{25})^2)^{-1}, \quad u \in [0,100] \quad (43)$$

$$\text{and} \quad \mu_{\text{TRUE}}(v) = v^2, \quad v \in [0,1] \quad (44)$$

then (41) yields

$$\Pi_{\text{Age}(\text{Bob})} = (1 + (\frac{u}{25})^2)^{-4} \quad (45)$$

as the possibility distribution of the age of Bob.

Used in combination, the translation rules stated above provide a system for the determination of the possibility distribution induced by a fairly complex composite proposition. For example, the proposition

If X is not very large and Y is more or less small then Z is very very large.

induces the conditional possibility distribution described by

$$\begin{aligned} \pi_{(Z|X,Y)}(w|u,v) &= 1 \wedge (1 - (1 - \mu_{\text{LARGE}}^2(u)) \\ &\quad \wedge \mu_{\text{SMALL}}(v) + \mu_{\text{LARGE}}^4(w)). \end{aligned} \quad (46)$$

It should be noted that rules of this type have found practical applications in the design of fuzzy logic controllers in steel plants, cement kilns and other types of industrial process control applications in which instructions expressed in a natural language are transformed into control signals [31-33].

Rules of Inference

In our approach to approximate reasoning, the translation rules for imprecise propositions serve as a preliminary to the application of various rules of inference to the possibility distributions which are induced by such propositions, leading to other possibility distributions which upon retranslation yield the conclusions which may be drawn from the premises.

More specifically, the basic rules of inference in FL are the following.

1. Projection rule. Consider a fuzzy proposition whose translation is expressed as

$$p \rightarrow \Pi(X_1, \dots, X_n) = F \quad (47)$$

and let $X_{(s)}$ denote a subvariable of the variable $X \triangleq (X_1, \dots, X_n)$, i.e.,

$$X_{(s)} = (X_{i_1}, \dots, X_{i_k}) \quad (48)$$

where the index sequence $s \triangleq (i_1, \dots, i_k)$ is a subsequence of the sequence $(1, \dots, n)$.

Furthermore, let $\Pi_{X_{(s)}}$ denote the marginal possibility distribution of $X_{(s)}$; that is,

$$\Pi_{X_{(s)}} = \text{Proj}_{U_{(s)}} F \quad (49)$$

where U_i , $i = 1, \dots, n$, is the universe of discourse associated with X_i ;

$$U_{(s)} = U_{i_1} \times \dots \times U_{i_k} \quad (50)$$

and the projection of F on $U_{(s)}$ is defined by the possibility distribution function

$$\begin{aligned} \pi_{X_{(s)}}(u_{i_1}, \dots, u_{i_k}) \\ = \sup_{u_{j_1}, \dots, u_{j_m}} \mu_F(u_1, \dots, u_n) \end{aligned} \quad (51)$$

where $s' \triangleq (j_1, \dots, j_m)$ is the index subsequence which is complementary to s , and μ_F is the membership function of F .

Now let q be a retranslation of the possibility assignment equation

$$\Pi_{X_{(s)}} = \text{Proj}_{U_{(s)}} F. \quad (52)$$

Then, the projection rule asserts that q may be inferred from p . In a schematic form, this assertion may be expressed more transparently

$$\begin{aligned} \text{as } p &\rightarrow \Pi(X_1, \dots, X_n) = F \\ q &\leftarrow \Pi_{X_{(s)}} = \text{Proj}_{U_{(s)}} F \end{aligned} \quad (53)$$

2. Conjunction rule. Consider a proposition p which is an assertion concerning the possible values of, say, two variables X and Y which take values in U and V , respectively. Similarly, let q be an assertion concerning the possible values of the variables Y and Z , taking values in V and W . With these assumptions, the translations of p and q may be expressed as

$$\begin{aligned} p &\rightarrow \Pi^p_{(X,Y)} = F \\ q &\rightarrow \Pi^q_{(Y,Z)} = G \end{aligned} \quad (59)$$

Let $\bar{F}$ and $\bar{G}$ be, respectively, the cylindrical extensions of F and G in $U \times V \times W$. Thus,

$$\bar{F} = F \times W \quad (60)$$

and

$$\bar{G} = U \times G. \quad (61)$$

Using the conjunction rule, we can infer from p and q a proposition which is defined by the following scheme (the reverse arrow $\leftarrow$ denotes retranslation, i.e., reverse translation):

$$r \rightarrow \Pi^p_{(X,Y)} = F \quad (62)$$

$$q \rightarrow \Pi^q_{(Y,Z)} = G \quad (63)$$

$$r \leftarrow \Pi_{(X,Y,Z)} = \bar{F} \cap \bar{G} \quad (64)$$

On combining the projection and conjunction rules, we obtain the compositional rule of inference (67) which includes the classical modus ponens as a special case.

More specifically, on applying the projection rule to (64), we obtain the following inference scheme

$$p \rightarrow \Pi^p_{(X,Y)} = F \quad (65)$$

$$q \rightarrow \Pi^q_{(Y,Z)} = G$$

$$r \leftarrow \Pi^r_{(X,Z)} = F \circ G$$

where the composition of F and G is defined by

$$\mu_{F \circ G}(u, w) = \sup_v (\mu_F(u, v) \wedge \mu_G(v, w)) \quad (66)$$

In particular, if p is a proposition of the form " X is F " and q is a proposition of the form "If X is G then Y is H ," then (65) becomes

$$p \rightarrow \Pi_X = F \quad (67)$$

$$q \rightarrow \Pi_{(Y|X)} = \bar{G}' \oplus \bar{H}$$

$$r \leftarrow \Pi_{(Y)} = F \circ (\bar{G}' \oplus \bar{H})$$

The rule expressed by (67) may be viewed as a generalized form of modus ponens which reduces to the classical modus ponens when $F = G$ and F, G, H are nonfuzzy sets.

Stated in terms of possibility distributions, the generalized modus ponens places in evidence the analogy between probabilistic and possibilistic inference. Thus, in the case of probabilities, we can deduce the probability distribution of Y from the knowledge of the probability distribution of X and the conditional probability distribution of Y given X . Similarly, in the case of possibility distributions, we can infer the possibility distribution of Y from the knowledge of the possibility distribution of X and the conditional possibility distribution of Y given X .

It is important to note that the generalized modus ponens as expressed by (67) may be used to enlarge significantly the area of applicability of rule-based systems of the type employed in MYCIN and other expert systems. This is due primarily to two aspects of (67) which are not present in conventional rule-based systems: (a) in the propositions " X is F " and "If X is G then Y is H ," F, G and H may be fuzzy sets; and (b) F and G need not be identical. Thus, as a result of (a) and (b), a rule-based system employing (67) may be designed to have an interpolative capability [6].

Note. Due to limitations on space, the sections dealing with credibility analysis and examples of approximate reasoning have been deleted. Full text of the paper is available on request.

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