

## AN APPLICATION OF DECISION ANALYSIS TO STRATEGY-MAKING IN GAME PLAYING

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This paper presents a game playing model of Score-Four that is a kind of alignment game similar to three dimensional tic-tac-toe. The present model aims at winning a game most efficiently. The principal proposal is an adoption of Decision Analysis, a wide-spread method of systems analysis, instead of Minimax Procedure. This paper includes a static evaluation on the state of game, a brief illustration of Decision Analysis, a method of strategy-making and an outline of a game playing machine. Based on the results from the experiments conducted with human players, we assert that Decision Analysis is a good method to make strategies which enable us to take advantage of the opponent's mistakes and it is shown that the risk caused by abandonment of the conventional assumption that the opponent selects the best move is averted by the introduction of a learning system.

### 1. INTRODUCTION

There are many papers as for game playing model which satisfies the following conditions.

1. Play starts from a situation which is not advantageous to either player.
2. The interest of game playing is just "to win a game" and there is no more.

If a game is played under the above conditions, it would be valid that a player adopts the assumption that the opponent selects the best of all legal moves in every situation, for he sometimes loses a game by making little of his opponent's skill. In this sense Minimax Procedure [1] on the basis of the above assumption is adequate for strategy-making when we need not dare to run a risk. However games are not always played under such conditions. For example, there are sometimes occasions when the interest of game playing is "to win a game more efficiently" rather than "to win a game". More efficient winning means winning with fewer moves in some games or with capturing more pieces of the opponent in other games, generally speaking, overwhelming one. In this case it becomes important for a player to make more efficient moves, which enable him to take advantage of his opponent's mistakes. In other words, the conventional assumption is too passive for winning a game efficiently. Similarly efficient moves should be played in occasions when a heavy handicap is imposed.

Decision Analysis [2] is effective for strategy-making in a game playing model where we are aiming at an efficient winning. In this method we can take into account the probabilities of its opponent's mistakes.

In this study, it is assumed that the opponent plays a move of the best three among all the possible moves even though he is an unskillful player. Then the key point of this method is the determination of the judgemental probabilities assigned to these three moves. We introduce an index representing the opponent's skill. It should be noted that there may be a risk caused by underestimation of the opponent's skill or inefficiency by overestimation. This problem, however, is resolved by introducing a learning system which improves the initial value of the index step by step by evaluating the opponent's moves.

We are applying the above method to strategy-making in Score-Four game and Score-Four Playing Machine, which is a system playing with a human player, is constructed.

### 2. DESCRIPTION OF SCORE-FOUR GAME

Score-Four is an alignment game played by two persons; one uses black balls (simply called Blacks) and the other white balls (called Whites) as their pieces. The game is played on a board with sixteen sticks which penetrate the balls through their small hole. (See Fig.1)

Since each stick is capable of accepting four balls, the board has 64 points on which players put them. Each point is denoted by a pair of number,  $(m, n)$ , where  $m$  and  $n$  mean a stick number and a horizontal plane number respectively, and  $1 \leq m \leq 16$ ,  $1 \leq n \leq 4$ .

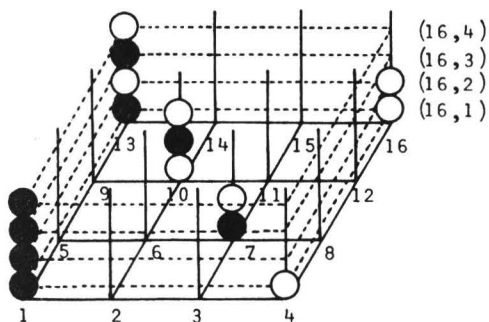


Fig.1 Board of Score-Four

Play starts with the first move of the player who has Blacks (called Player Black) and then each plays a move by turns. Players, however, can not put their balls on arbitrary points, that is, they are allowed to make a move on  $(m, n)$  only if  $n$  is equal to 1 or  $(m, n-1)$  is occupied. Such a point is called legal. In Fig.1, for example,  $(16, 3)$  is a legal point and  $(16, 4)$  is an illegal one. The game is won when a player forms a string of his four balls in any line, either horizontally, vertically or diagonally. If it is not won within 64 moves, it ends in a draw.

Now, we present some important properties of the game, which will help us to look for factors to evaluate a board situation.

1. The game is played on a three-dimensional board.
2. The game ends within 64 moves.
3. The number of legal moves in a board situation is equal to or smaller than 16.
4. The number of lines in which it is possible to form a string of four balls is 76.

### 3. METHOD OF EVALUATING BOARD SITUATIONS

A situation of the board may be determined with the consideration on all the possible states of 76 lines. Therefore we adopt them as the fundamental factors to evaluate a board situation. Each line is in one of the following eight types of states. Let  $\alpha_j$  ( $j=1, \dots, 8$ ) be an index for evaluating a line in  $j$ -th state.

STATE I (Four) : There is a string of four Blacks (Whites). We fix the index,  $\alpha_1 = 150$ .

STATE II (Active-Three) : Three points are occupied by Blacks (Whites) and the remainder is unoccupied and legal. If player Black (White) makes the next move and he can win a game by forming "Four", then  $\alpha_2 = 70$ , otherwise  $\alpha_2 = 10$ . (See Fig.2, a)

STATE III (Sleeping-Three) : Blacks (Whites) occupy three points in the line and the remainder is unoccupied and illegal.  $\alpha_3 = 18$ . (Fig.2, b)

STATE IV (Active-Two) : There are two points occupied by Blacks (Whites) and two vacant and legal points.  $\alpha_4 = 4$ . (Fig.2, c)

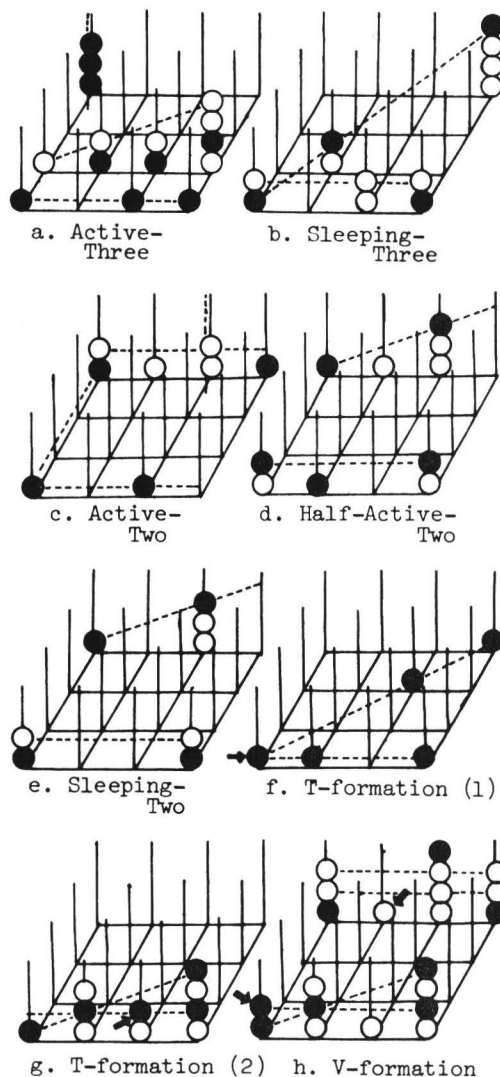


Fig.2 Factors to Evaluate a Board Situation

STATE V (Half-Active-Two) : There are two Blacks (Whites) and two vacant points; one is legal and the other is illegal. At his next turn, Player Black (White) can form "Sleeping-Three".  $\alpha_5 = 5$ . (Fig.2, d)

STATE VI (Sleeping-Two) : There are two Blacks (Whites) and two unoccupied and illegal points. Player Black (White) can not form "Active-Three" nor "Sleeping-Three" at his next turn.  $\alpha_6 = 8$ . (Fig.2, e)

STATE VII : There is only a Black (White) and the others are unoccupied.  $\alpha_7 = 1$ .

STATE VIII : Both Blacks and Whites exist or there is no ball in the line.  $\alpha_8 = 0$ .

Now we show the two important patterns on which games are won.

1. One forms two or more "Active-Three" with a move. He can surely win the game at his next move. This pattern is called "T-formation". (See Fig.2, f, g).
2. One player forms "Active-Three" where the only one vacant point is just under the vacant of his "Sleeping-Three". He can win the game at his next move. We call it "V-formation" which is shown in Fig.2, h.

So, as an additional factor, we consider whether or not there exists T- or V-formation and let  $\alpha_9 = 50$ .

Let us now show how to evaluate each factor as stated above. Let  $u_i$  ( $i=1,2, \dots, 76$ ) and  $v$  be the evaluations of  $i$ -th line and T- or V-formation, respectively. Then,

$$u_j = (-1)^k \alpha_j : \text{if } i\text{-th line belongs to STATE } j. \quad (1)$$

$(i=1, \dots, 76) (j=1, \dots, 8)$

$$v = \begin{cases} (-1)^k \alpha_9 : \text{if there exists T- or V-formations of one side.} \\ 0 : \text{otherwise.} \end{cases} \quad (2)$$

where

$$k = \begin{cases} 0 : \text{if the factor is evaluated for Score-Four Playing System} \\ 1 : \text{otherwise.} \end{cases}$$

Then, static evaluation,  $U$ , of a board situation is defined as the sum of  $u_i$  ( $i=1, \dots, 76$ ) and  $v$  as :

$$U(S) = \sum_{i=1}^{76} u_i + v. \quad (3)$$

#### 4. STRATEGY-MAKING BY DECISION ANALYSIS

First, we make a brief explanation of Decision Analysis. When one intends to make his decision about a certain problem by using Decision Analysis, he must generate the decision tree as shown in Fig.3. It represents a chronological arrangement of choices controlled by him and chance. The tree graph is composed of chance points, strategies selected by him, events dependent on chance and utilities. In the above, it is only at decision points that decision maker can decide which path he goes to. The main procedures of Decision Analysis are "folding back" at decision points and "averaging out" at chance points. In Fig.3, folding back at  $C_1$  is the procedure to assign the maximum of  $u_1$  and  $u_2$  to  $C_1$  and averaging out at  $C_2$  means to assign the expected value of  $u_3$  and  $u_4$  to  $C_2$ . After he accomplishes the procedures at every points, he only has to select the strategy leading to the point where the expected utility is the largest.

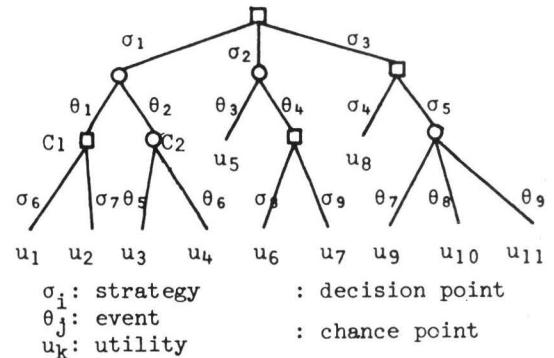


Fig.3 Decision Tree

In game playing a player can not predict the opponent's next move precisely. He can consider at best the probabilities in terms of judgemental probability. Therefore we may take the opponent's next move as an event dependent on chance.

Now let us show a way to calculate the judgemental probabilities,  $P_i$ , that the opponent plays the  $i$ -th best move,  $m_i$ . Suppose there are  $n$  legal moves at his turn. Then, first, the judgemental probabilities must satisfy the following equation :

$$\sum_{i=1}^n P_i = 1. \quad (4)$$

It is a natural assumption that the judgemental probabilities depend on the opponent's skill, because skillful player is likely to play the best move at a high probability and on the other

hand beginner frequently plays a worse move. Let by  $f_i$  be denoted an index for evaluating  $i$ -th best move where  $f_1 \geq f_2 \geq \dots \geq f_n$ . Then the degree of the opponent's skill may be roughly represented as follows;

$$\bar{f} = \sum_{i=1}^n P_i f_i \quad (5)$$

where  $\bar{f}$  means the expected value of the evaluation of the opponent's next move. Furthermore, the concept of entropy is utilized to estimate the judgemental probabilities. Entropy in information theory [3] implies a degree of "uncertainty" or "randomness" of events. If we choose  $P_i$  ( $i=1, \dots, n$ ) so as to maximize the entropy, they are the most reasonable judgemental probabilities. It is because the maximization of uncertainty implies the safest or most temperate estimation.

Entropy,  $H$ , is defined as:

$$H = - \sum_{i=1}^n P_i \log_2 P_i \quad (6)$$

By maximizing  $H$  subject to Eqs. (4) and (5), we obtain:

$$P_i / P_{i+2} = (P_i / P_{i+1})^{K_i} \quad (i=1, \dots, n-2) \quad (7)$$

where

$$K_i = \frac{f_i - f_{i+2}}{f_i - f_{i+1}}$$

Here, let us assume that the opponent plays only one of the best three moves  $m_1$ ,  $m_2$  and  $m_3$ . Then, from Eqs. (4), (5) and (7) we readily obtain the following equation:

$$(f_1 - \bar{f}) + (f_2 - \bar{f})x + (f_3 - \bar{f})x^K = 0 \quad (8)$$

where  $x = P_2 / P_1$  and  $K = (f_1 - f_3) / (f_1 - f_2)$ .

If Eq.(8) has zero or positive real solution, judgemental probabilities are expressed as follows. (See APPENDIX I)

$$\begin{aligned} P_1 &= \frac{1}{1 + x + x^K} \\ P_2 &= \frac{x}{1 + x + x^K} = P_1 x \\ P_3 &= \frac{x^K}{1 + x + x^K} = P_1 x^K \end{aligned} \quad (9)$$

Since it is not guaranteed that Eq.(8) has zero or a positive real solution, we show ways determine  $f_1$ ,  $f_2$ ,  $f_3$  and  $\bar{f}$  and examine the solvability. First, consider the case where  $K=2$ .

In this case one may find that  $f_1$ ,  $f_2$  and  $f_3$  are an arithmetical progression and that  $P_1$ ,  $P_2$  and  $P_3$  are a geometrical one. Moreover, except for the case where  $\bar{f} = f_3$ , Eq.(8) comes to be a quadratic one which has two real solutions - one of them is either positive or zero and the other is negative - because we are assuming that the opponent plays one of the best three moves, that is,

$$f_3 \leq \bar{f} \leq f_1 \quad (10)$$

Therefore, if  $\bar{f}$  takes on any value between zero and one, it is estimated that  $f_1 = 1.0$ ,  $f_2 = 0.5$  and  $f_3 = 0$ . Then in the case where  $\bar{f} = f_3 = 0$ , we obtain that  $P_1 = P_2 = 0$  and  $P_3 = 1$  from Eqs.(4), (5) and (7).

However it seems to be more natural than the above to consider that evaluation,  $f_i$ , for  $i$ -th move,  $m_i$ , is intimately related to the board situation,  $S$ , or to its evaluation,  $U(S)$ , after  $m_i$  is played. This idea is appreciated by recalling that the facts that we sometimes find both the extreme cases where  $m_1$  and  $m_2$  have little different evaluations and where a player loses because he has played  $m_2$  instead of  $m_1$ . Let  $S_i$  be a board situation after a move,  $m_i$ , is played. Then, we define  $f_i$  by using  $U(S_i)$ -s given in Eq.(3) as follows;

$$f_i = \frac{U(S_i) - U(S_n)}{U(S_1) - U(S_n)} \quad (11)$$

where  $S_1$  and  $S_n$  mean the board situations after the best move and the worst one are played, respectively.

In general, it is impossible to solve Eq.(8) analytically. But if  $\bar{f}$  takes any value between  $f_3$  and 1, it is easily solved by using a computer. (See APPENDIX II) On the occasion when  $\bar{f} < f_3$ , Eq.(8) does not have any positive solution. It is, however, so risky to make little of the opponent's skill that we consider as if  $\bar{f} = f_3$  from the view point of safety playing.

Now it is necessary to consider how to estimate  $\bar{f}$ . Since it is given without any reliable foundation at the beginning of games, it should be improved during the game. In this study we adopt the next equation for this purpose. Let  $\bar{f}^k$  and  $\bar{f}^k$  be the evaluation of the move,  $m^k$ , played by the opponent at his  $k$ -th turn and the estimation of  $\bar{f}$  at the moment just after  $m^k$  is played, respectively.

$$\bar{f}^k = (1 - \beta) \bar{f}^{k-1} + \beta \bar{f}^k \quad (k=1, \dots, 32) \quad (12)$$

where  $\beta$  is a constant parameter which takes on a value between zero and one. The above is the major algorithm to be installed by SFPM standing for Score-Four Playing Machine. In SFPM, the depth of lookahead is considered to be only two steps but it is almost enough on usual occasions, and since the static evaluation function takes into account T- and V-formations which guarantee a winning several moves ahead, the actual depth corresponds to four steps or more in the critical moment. The further details on SFPM will be presented at oral presentation.

##### 5. EXPERIMENTS : SFPM V.S. HUMAN PLAYERS

The experiments are conducted to investigate the advantages of the method presented above. Experimental results of game playing with human players are shown in Table 1. The number of the subjects is fifteen and each plays several games under different conditions. Average number (A.N.) of moves required for SFPM's winning implies the index of efficiency of winning. As for conditions,  $\bar{f}^0$  is the initial estimation of the opponent's skill. The conditions that  $\bar{f}^0 = 1.0$  and  $\bar{f}^0 = 0.5$  mean the overestimation and underestimation respectively, because  $\bar{f}^k$  of every subject took a value between 0.65 and 0.93 at the end of a game. Fig.4 shows a typical example of the fluctuations of  $\bar{f}^k$  observed during a game.

Table 1. Results of Experiments.

| Experiment Number | Conditions |             | Results of Games     | A.N. |
|-------------------|------------|-------------|----------------------|------|
|                   | $\beta$    | $\bar{f}^0$ |                      |      |
| I                 | 0          | 1.0         | 7 wins and 1 defeat  | 43   |
| II                | 0          | 0.8         | 5 wins and 3 defeats | 28   |
| III               | 0          | 0.5         | 4 wins and 4 defeats | 35   |
| IV                | 0.05       | 1.0         | 6 wins and 2 defeats | 39   |
| V                 | 0.05       | 0.8         | 5 wins and 3 defeats | 29   |
| VI                | 0.05       | 0.5         | 6 wins and 2 defeats | 26   |

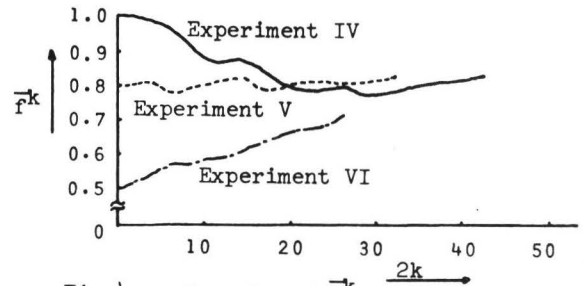


Fig.4 Example of  $\bar{f}^k$  in Game Playing

Human player in this example belongs to middle class and he lost in the three experiments shown in Fig.4, but he won in Experiment III at the 34th move.

Next it is shown how SFPM plays a move in accordance with  $\bar{f}^k$ . When  $\bar{f}^k$  is high, that is, SFPM regards its opponent as a skillful player, it inclines to form "Sleeping-Two" and "Sleeping-Three" rather than "Active-Two", "Half-Active-Two" and "Active-Three". This tendency is usually found in games played by skillful players. The tactics are adequate to game playing with skillful players for the following reasons.

1. Skillful players seldom miss blocking their opponent's "Active-Three".
  2. To form "Sleepin-Two" or "Sleeping-Three" makes the opponent disadvantageous gradually because they restrict the opponent's selection of moves.
  3. "Sleeping-Three" is a necessary preliminary for T- and V-formations. (Refer to Fig.2)
- The tactics are, however, not efficient to defeat a beginner or an unskillful player in the sense that many moves are required to win a game, because they are the protracting tactics. On the other hand, when  $\bar{f}^k$  is small, SFPM inclines to form "Active-Two", "Half-Active-Two" and "Active-Three" rather than "Sleeping-Two" and "Sleeping-Three". These tactics are effective for game playing with unskillful players who sometimes miss blocking the opponent's "Active-Three". However they are too simple to win when the opponent is skillful. We can say from the experimental results that the figures in Experiment IV, V and VI are much improved in comparison with Experiments I, II and III. The strategy-making in Experiment I is the same as the one by Minmax Procedure. It should be noticed that, in order that the use of Decision Analysis works well, it is necessary that the opponent's skill is known rather precisely.

## 6. CONCLUSIONS

In this paper we have presented a game-playing-machine which employs Decision Analysis as a substitutional method for Minmax Procedure. A game-playing-machine controlled by Minmax Procedure can not cope with the game playing in which efficiency of winning is make much of, while it works very well if it aims at only winning, in more strict words, minimizing the risk of losing. Decision Analysis is a good method to decide strategies which enable a game playing machine to take advantage of its opponent's mistakes -not playing the best move- and to win a game efficiently, while the good efficiency is obtained at the expense of increase in the risk. However, such a risk is capable of being averted to passable extent by estimating the opponent's skill correctly and assuming that he plays a move of the best three in any board situation. Learning system yields the high rate of winning and the good efficiency even though the opponent's skill is misestimated at the beginning of a game.

Besides, we may conclude that Decision Analysis is also useful for many other games which do not satisfy the conditions stated in INTRODUCTION. Finally we suggest that the opponent's move selection is usually influenced by not only his skill but also his various biases; some players may incline to make such moves that are very offensive or very defensive [4]. If we take the bias into consideration in order to determine the judgemental probabilities, Decision Analysis must be a more useful method of strategy-making in game playing.

## ACKNOWLEDGEMENTS

The authors wish to thank Dr. M. Sugeno and Mr. S. Kainuma for their perceptive comments and suggestions.

## APPENDIX I

Let us transform Eq.(7), then we obtain:

$$P_3 = P_2^K P_1^{1-K}.$$

Since  $x = P_2/P_1$ , we can re-write it as:

$$P_3 = (P_1 x)^K P_1^{1-K} = P_1 x^K.$$

Therefore

$$P_1 + P_2 + P_3 = P_1(1 + x + x^K) = 1,$$

which directly leads to Eq.(9).

## APPENDIX II

Let us denote Eq.(8) by  $F(x) = 0$ .

1. If  $f_3 < \bar{f} \leq f_1$  and  $f_3 < f_2 < f_1 \equiv 1$ , then

$$F(0) = f_1 - \bar{f} \geq 0$$

and

$$\frac{d^2 F(x)}{dx^2} = K(K-1)(f_3 - \bar{f})x^{K-2} < 0 \quad (x > 0).$$

Therefore Eq.(8) has only a real solution within the interval,  $[0, \infty)$  and we can obtain the judgemental probabilities from Eq.(9).

2. If  $\bar{f} = f_3$  and  $f_3 < f_2 < f_1 \equiv 1$ , then

we obtain  $P_1 = P_2 = 0$ , and  $P_3 = 1$  from Eqs.(4), (5) and (7).

3. If  $f_3 \leq \bar{f} \leq f_1 \equiv 1$ , and  $f_i = f_{i+1}$  ( $i = 1, 2$ ),

then we can calculate the judgemental probabilities from Eqs.(4) and (5).

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