

RESOLUTION IN A NEW MODAL LOGIC

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This paper proposes a new logic called manner logic(ML) and considers resolution techniques for mechanical theorem proving in the ML. The ML deals with statements which contain adverbs or verbs representing manner such as possibility, uncertainty and causality. Manner is expressed by a manner operator "*", which is a monadic operator. By using manner operators, a postulate can be four-valued; true(T), true in the sense of "*" (T*), false in the sense of "*" (F*), or false(F). In the ML, an atomic formula is assigned a value T or F, but a well-formed formula is assigned a value T, T*, F*, or F. Therefore, the ML can represent more complicated statements than the ordinal predicate logic. Its logical manipulation is quite simple, since it can be considered an extension of the ordinal binary logic. This paper also discusses resolution techniques in a refutation process. In our method, an additional valid clause is used in a refutation process to resolve clauses with manner operators which cannot be resolved directly.

1. INTRODUCTION

In the real world, the situation is often encountered where a postulate can be many-valued rather than two-valued. However, the many-valued logic is too complex to be applied to predicate calculus in artificial intelligence.

There are some other interesting logics such as a modal logic, tense logic or fuzzy logic. The modal logic [2], [3] had its effective beginning with work of C.L. Lewis[3] some sixty years ago, and is known as the logic of necessity and possibility. The tense logic [4], [6] deals with statements containing tensed verbs or explicit temporal references, and is logically similar to the modal logic. In these logics, the mode and tense are represented by two monadic symbols, and a postulate can be two-valued; true or false.

The present paper proposes a new logic which falls under the heading of the ordinal modal logic in a wide sense. The proposed logic deals with statements which contain adverbs or verbs representing manner such as possibility, uncertainty and causality. Monadic operators called manner operators are introduced to express such manner. A postulate in the logic can be four-valued; for example,

true, possibly true, possibly false, or false. That is, a value T or F is assigned to each atomic formula in a well-formed formula(wff), but a value T, T*, F*, or F is assigned to the wff as described in the following sections. Consequently, we call the new logic the multi-binary logic or manner logic(ML). The manner logic can represent more complicated statements than the ordinal predicate logic, since it is essentially a four-valued logic.

Thus we are interested in the application of the ML to solve intellectual difficult problems. As is well known, the resolution principle due to Robinson[7], [8], [9] is a simple but extremely useful technique for mechanical theorem proving. It is difficult, however, to apply the Robinson's resolution principle to the ML, because clauses contain manner operators. To circumvent this difficulty, additional clauses which are valid are used in a refutation process. The present paper considers some fundamental properties of the ML, and gives a new rule of inference for mechanical theorem proving.

2. FUNDAMENTAL PROPERTIES

This section is devoted to consideration of fundamental properties of the ML. We shall introduce the symbol "*" with intention of interpreting it as a manner operator, which is placed on the shoulder of well-formed formulas. Suppose a proposition P is interpreted as "She is beautiful". Then P* is "She is possibly beautiful", "She will be beautiful", or "She seems to be beautiful" or "She is very beautiful", in the sense of the modal, tense or fuzzy logic, respectively.

The symbols such as P, Q and R, that are used to denote propositions are called atoms. Definition 1 Well-formed formulas (wff) or formulas for short in the ML are defined recursively as follows:

1. An atom is a formula.
2. If G is a formula, then so are $\sim G$ and G^* .
3. If G and H are formulas, then so are $G \vee H$, $G \wedge H$, $G \rightarrow H$.
4. All formulas are generated by applying the above rules.

In the ordinal logic, a well-formed formula can be either true or false. Here we represent "true" by T and "false" by F. Therefore, $\sim G$ is T when G is F, and is F when G is T. In the similar manner, G^* is T* when G is T, and is F* when G is F. Consequently, a well-formed formula G can be T, T*, F* or F; that is, true, true in the sense of "*", false in the sense of "*", or false. For example, G is true, G is probably true, G is probably false, or G is false.

We begin by defining some logical symbols used here by means of Table 1. They are $\sim$ (negation), * (manner), $\vee$ (disjunction), $\wedge$ (conjunction), and $\rightarrow$ (implication). This table expresses the truth value of the statements. It is seen from Table 1 that the implication can be represented by a negation and disjunction as follows:

$$X \rightarrow Y = \sim X \vee Y$$

where X and Y are wffs.

Some examples of statements containing manner operators are shown below. Let P and Q be propositional symbols interpreted as

P : She is young.

Q : She is beautiful.

Then the expression $X \rightarrow Y$ asserts as follows:

$P \rightarrow Q$: If she is young, then she is beautiful.

P	F	F	T	T
Q	F	T	F	T
$\sim P$	T	T	F	F
P^*	F*	F*	T*	T*
$(P^*)^*$	F*	F*	T*	T*
$\sim(P)^*$	T*	T*	F*	F*
$(\sim P)^*$	T*	T*	F*	F*
$P \vee Q$	F	T	T	T
$P \vee Q^*$	F*	T*	T*	T*
$P^* \vee Q$	F*	T	T*	T
$P^* \vee Q^*$	F*	T	T	T*
$P \wedge Q$	F	F	F	T
$P \wedge Q^*$	F	F	F*	T*
$P^* \wedge Q$	F	F*	F	T*
$P^* \wedge Q^*$	F*	F	F	T*
$P \rightarrow Q$	T	T	F	T
$P \rightarrow Q^*$	T	T	F*	T
$P^* \rightarrow Q$	T*	T	F*	T
$P^* \rightarrow Q^*$	T	T*	F	F

Table 1 Truth table

- $P \rightarrow Q^*$: If she is young, then she seems to be beautiful.
 $P^* \rightarrow Q$: If she seems to be young, then she is beautiful.
 $P^* \rightarrow Q^*$: If she seems to be young, then she seems to be beautiful.
 $(P \rightarrow Q)^*$: It seems that if she is young she is beautiful.

As shown in the above examples, more complex statements can be represented if manner operators are employed.

Now, we consider the fundamental properties of the ML. Let P, Q, and R be atoms, and X, Y, and Z be well-formed formulas. From the truth table, Table 1, we have the following laws:

Idempotent laws
 $X \vee X = X, X \wedge X = X$ (1)

Absorption laws
 $X \vee (X \wedge Y) = X, X \wedge (X \vee Y) = X$ (2)

Commutative laws
 $X \vee Y = Y \vee X, X \wedge Y = Y \wedge X$ (3)

Associative laws
 $X \vee (Y \vee Z) = (X \vee Y) \vee Z,$
 $X \wedge (Y \wedge Z) = (X \wedge Y) \wedge Z$ (4)

Distributed laws
 $X \vee (Y \wedge Z) = (X \vee Y) \wedge (X \vee Z),$
 $X \wedge (Y \vee Z) = (X \wedge Y) \vee (X \wedge Z)$ (5)

Involution laws

$$\sim(\sim X) = X. \quad (6)$$

De Morgan's laws

$$\begin{aligned} \sim(X \vee Y) &= \sim X \wedge \sim Y, \\ \sim(X \wedge Y) &= \sim X \vee \sim Y. \end{aligned} \quad (7)$$

Contradiction laws

$$X \vee \sim X = T, \quad X \wedge \sim X = F. \quad (8)$$

Complement laws

$$\begin{aligned} X \vee F &= X, & X \wedge T &= X \\ X \wedge F &= F, & X \vee T &= T. \end{aligned} \quad (9) \quad (10)$$

These laws are the same as those in the ordinal logic, but the following laws are peculiar to the ML.

$$\circ (P^*)^* = P^*. \quad (11)$$

$$\circ \sim(P^*) = (\sim P)^*. \quad (12)$$

$$\begin{aligned} P^* &= (P \wedge T^*) \vee (\sim P \wedge F^*), \\ (\sim P)^* &= (P \vee T^*) \wedge (\sim P \vee F^*). \end{aligned} \quad (13)$$

$$\circ (X \vee Y)^* = (X^* \vee Y^*) \wedge T^* \vee (X^* \wedge Y^* \wedge F^*). \quad (14)$$

$$\circ (X \wedge Y)^* = (X^* \wedge Y^* \vee F^*) \wedge (X^* \vee Y^* \vee T^*). \quad (15)$$

Using the laws (1) - (15), we have the logical equivalences as follows:

$$\circ P \wedge P^* = P \wedge T^*, \quad \sim P \wedge \sim P^* = \sim P \wedge T^*, \quad (16)$$

$$\circ P \wedge \sim P^* = P \wedge F^*, \quad \sim P \wedge P^* = \sim P \wedge F^*.$$

[Proof] The proof is obtained from Eq. (13).

$$\circ P \vee P^* = P^* \vee F^*, \quad \sim P \vee \sim P^* = \sim P^* \vee F^*, \quad (17)$$

$$\circ P \vee \sim P^* = P \vee T^*, \quad \sim P \vee P^* = \sim P \vee T^*.$$

[Proof] The proof is obtained from Eq. (13).

$$\circ (\sim P \vee P^*) \wedge (P \vee \sim P^*) = T^*, \quad (18)$$

$$\circ \sim P \wedge P^* \vee P \wedge \sim P^* = F^*.$$

[Proof] The proof is obtained from Eqs. (16) and (17).

$$\circ (P \wedge P^*) \vee (\sim P \wedge \sim P^*) = T^*, \quad (19)$$

$$\circ (P \vee P^*) \wedge (\sim P \vee \sim P^*) = F^*.$$

[Proof] The proof is obtained from Eqs. (16) and (17).

$$\circ (P \vee Q)^* \wedge (\sim P \vee \sim Q)^* = (P \wedge \sim Q \vee \sim P \wedge Q) \wedge T^*, \quad (20)$$

$$\circ (P \vee \sim Q)^* \wedge (\sim P \vee Q)^* = (P \wedge Q \vee \sim P \wedge \sim Q) \wedge T^*.$$

[Proof] The proof is obtained from Eqs. (14) and (15).

$$\circ (P^* \wedge \sim Q^*) \vee (\sim P^* \wedge Q^*) = (P \wedge \sim Q) \vee (\sim P \wedge Q),$$

$$\circ (P^* \wedge Q^*) \vee (\sim P^* \wedge \sim Q^*) = (P \wedge Q) \vee (\sim P \wedge \sim Q). \quad (21)$$

[Proof] The proof is obtained from Eqs. (14) and (15).

$$\circ (P^* \vee Q)^* = (P^* \vee Q^*)^* = (P \vee Q)^*, \quad (22)$$

$$\circ (P^* \wedge Q)^* = (P^* \wedge Q^*)^* = (P \wedge Q)^*.$$

[Proof] The proof is obtained from Eqs. (14) and (15).

$$\circ P \wedge Q \wedge (P \wedge Q)^* = P \wedge Q \wedge P^* \wedge Q^* = P \wedge Q \wedge T^*, \quad (23)$$

$$\circ P \vee Q \vee (P \vee Q)^* = P \vee Q \vee P^* \vee Q^* = P \vee Q \vee F^*.$$

[Proof] The proof is obtained from Eqs. (14) and (15).

$$\begin{aligned} \circ P^* &= P^* \wedge (P \vee F^*) = P^* \wedge (\sim P \vee T^*) \\ &= P^* \vee P \wedge T^* = P^* \vee \sim P \wedge F^*, \end{aligned} \quad (24)$$

$$\begin{aligned} \circ \sim P^* &= \sim P^* \vee \sim P \wedge T^* = \sim P^* \vee P \wedge F^* \\ &= \sim P^* \wedge (\sim P \vee F^*) = \sim P^* \wedge (P \vee T^*). \end{aligned}$$

[Proof] The proof is obtained from Eqs. (18) and (19).

$$\circ (P \rightarrow Q)^* = (P^* \rightarrow Q^*) \wedge (P \rightarrow Q^*) \wedge (P^* \rightarrow Q). \quad (25)$$

[Proof] The proof is obtained from Eqs. (14) and (15).

Now, we will show that a formula Y can be represented in the conjunctive and/or disjunctive normal form.

Definition 2. A formula Y is said to be in a conjunctive or disjunctive normal form if and only if Y has the form

$$Y = Y_1 \wedge \dots \wedge Y_n$$

$$\text{or} \quad Y = Y_1 \vee \dots \vee Y_n,$$

where each of $Y_1, \dots, Y_n$ is a disjunction or conjunction of literals with and without manner operators, respectively.

Let $P_1, P_2, \dots, P_m, Q_1, Q_2, \dots, Q_n$ be atoms, and $Y(P_1, P_2, \dots, P_m, Q_1^*, Q_2^*, \dots, Q_n^*)$ be an $(m+n)$ -adic formula.

Then we have

$$\begin{aligned} &Y(P_1, P_2, \dots, P_m, Q_1^*, Q_2^*, \dots, Q_n^*) \\ &= P_1 \wedge Y(1, P_2, \dots, P_m, Q_1^*, Q_2^*, \dots, Q_n^*) \\ &\quad \vee \sim P_1 \wedge Y(0, P_2, \dots, P_m, Q_1^*, Q_2^*, \dots, Q_n^*) \\ &\quad \vee \dots \dots \dots \\ &= (P_1 \wedge P_2 \wedge \dots \wedge P_m \wedge Q_1 \wedge Q_2 \wedge \dots \wedge Q_n \\ &\quad \wedge Y(1, \dots, 1, 1^*, \dots, 1^*)) \\ &\quad \vee \dots \dots \dots \end{aligned}$$

$$\begin{aligned} &\vee (\sim P_1 \wedge \sim P_2 \wedge \dots \wedge \sim P_m \wedge \sim Q_1 \wedge \sim Q_2 \wedge \dots \wedge \sim Q_n \\ &\quad \wedge Y(0, \dots, 0, 0^*, \dots, 0^*)). \end{aligned} \quad (26)$$

$$\begin{aligned}
& Y(P_1, P_2, \dots, P_m, Q_1^*, Q_2^*, \dots, Q_n^*) \\
& = (P_1 \vee Y(0, P_2, \dots, P_m, Q_1^*, Q_2^*, \dots, Q_n^*) \\
& \quad \wedge (\sim P_1 \vee Y(1, P_2, \dots, P_m, Q_1^*, Q_2^*, \dots, Q_n^*))) \\
& = (P_1 \vee \dots \vee P_m \vee Q_1 \vee \dots \vee Q_n \\
& \quad \vee Y(0, \dots, 0, 0^*, \dots, 0^*)) \\
& \quad \wedge \dots \dots \dots \\
& \quad \wedge (\sim P_1 \vee \dots \vee \sim P_m \vee \sim Q_1 \vee \dots \vee \sim Q_n \\
& \quad \quad Y(1, \dots, 1, 1^*, \dots, 1^*)), \\
& \hspace{15em} (27)
\end{aligned}$$

The former is the disjunctive normal form and the latter is the conjunctive normal form. As mentioned before, $Y(\dots)$ can be T, T*, F* or F, and then T* and F* appear in Eqs. (26) and (27). Using Eqs. (18) and (19), however, we can eliminate T* and F*. Therefore, any formula can be represented by using $\sim$, $*$, $\vee$, and $\wedge$.

3. RESOLUTION TECHNIQUES

As is well known, the resolution principle is a useful technique for making deduction in the system of the first-order predicate calculus. Here we consider techniques based on the resolution principle for the ML system. Some important definitions of terminology are given as follows:

Definition 3 A formula G is said to be satisfiable or semi-satisfiable if and only if there exists an interpretation I such that Y is evaluated to T or T*, respectively. In this case, we say I satisfies or semi-satisfies Y.

Definition 4 A formula is said to be unsatisfiable or semi-unsatisfiable, if and only if it is F or F* under all its interpretation. A formula is said to be satisfiable or semi-satisfiable if and only if it is not unsatisfiable or not semi-unsatisfiable, respectively.

Definition 5 A formula G is said to be a logical consequence or semi-consequence of a given set of formulas S, if and only if every interpretation satisfying or semi-satisfying S also satisfies or semi-satisfies G, respectively.

Definition 6 Given a formula G, a formula $\bar{G}$ obtained by removing all manner operators in G is said to be a reduced formula of G.

Definition 7 Given a clause C, a clause $\bar{C}$ obtained by removing all manner operators in C is said to be a reduced clause of C.

Definition 8 A clause which takes F or F* is said to be an empty or semi-empty clause,

respectively.

Processes for showing the unsatisfiability or semi-unsatisfiability are called refutation processes. The following theorems give basic properties used for refutation processes.

Theorem 1

The following clauses C_1 and C_2 are valid.

$$C_1 : (\sim P \wedge P^*) \rightarrow F^*$$

$$C_2 : (P \wedge \sim P^*) \rightarrow F^*.$$

[Proof]

From Eq. (17), we have

$$\begin{aligned}
(\sim P \wedge P^*) \rightarrow F^* &= P \vee \sim P^* \vee F^* \\
&= T,
\end{aligned}$$

which gives the proof.

q.e.d.

Theorem 2

Given formulas $Y_1, \dots, Y_n$ and a formula G, G is a logical consequence or semi-consequence of $Y_1, \dots, Y_n$ if and only if the formula $(Y_1 \wedge \dots \wedge Y_n \wedge \sim G)$ is unsatisfiable or semi-unsatisfiable, respectively.

[Proof]

($\Rightarrow$) Suppose G is a logical consequence or semi-consequence. In an interpretation I satisfying or semi-satisfying $Y_1, \dots, Y_n$, G is T or T* and then $Y_1 \wedge \dots \wedge Y_n \wedge \sim G$ is F or F*, respectively.

($\Leftarrow$) Suppose $Y_1 \wedge \dots \wedge Y_n \wedge \sim G$ is unsatisfiable or semi-unsatisfiable. For any interpretation I, if $Y_1 \wedge \dots \wedge Y_n$ is T or T*, then G must be T or T* in I. Therefore, G is a logical consequence or semi-consequence of $Y_1, \dots, Y_n$.

Now, we consider the application of the resolution principle due to Robinson to the theorem proving in the ML.

The resolution principle is essentially an elegant technique to check whether a given set of clauses is unsatisfiable or semi-unsatisfiable.

Consider the following set of clauses

$$S_1 = \{C_1, C_2, C_3\}$$

$$C_1 : P$$

$$C_2 : P \rightarrow Q^* \quad \text{or} \quad \sim P \vee Q^*$$

$$C_3 : \sim Q.$$

The resolution technique is very similar to that of Robinson except that clauses may contain manner operators. In this case, the resolvent Q^* (clause C_4) is obtained by deleting the pair of literals P and $\sim P$ from clauses C_1 and C_2 :

$$C_4 : Q^* \quad \text{from } C_1 \text{ and } C_2.$$

However, we have no way to get the resolvent from clauses C_3 and C_4 . Thus, a new set of clauses S_2 is introduced such that

$$S_2 = S_1 \cup C_5 \\ = \{C_1, C_2, C_3, C_5\}$$

$$C_5 : Q \vee \sim Q^* \vee F^*.$$

Since C_5 is a tautology from Theorem 1, S_2 is equivalent to S_1 from the point of view of refutation processes. Thus we have

$$C_6 : \sim Q^* \vee F^* \quad \text{from } C_3 \text{ and } C_5 \\ C_7 : F^* \quad \text{from } C_4 \text{ and } C_6.$$

Therefore, S_2 is semi-unsatisfiable and so is S_1 .

In the above example, however, if $C_3^1 = \sim Q^*$ is used instead of C_3 , then we have the empty clause as the resolvent of C_3^1 and C_4 .

Here a question arises : In what case a set of clauses is semi-unsatisfiable ? As described before, a formula G is a logical semi-consequence of $Y_1, \dots, Y_n$ if a semi-empty clause is obtained as a resolvent in a refutation process. This means, therefore, that G is true in a sense of " $\sim$ ". For example, consider the case where " G^* " asserts "It possibly rains". If a semi-empty clause is obtained, the proof is possibly true or the inference is possibly correct. That is, it is possibly true to conclude that it rains. On the other hand, the proof is true if an empty clause is obtained. The following sets of clauses are also semi-unsatisfiable and give the same results as above.

$$\{P, \sim P \vee Q, \sim Q^*\} \\ \{P^*, \sim P \vee Q, \sim Q\} \\ \{P, (\sim P \vee Q)^*, \sim Q\}.$$

Now we will show some important properties of a resolvent in the following theorems.

Theorem 3

A resolvent of given two clauses C_1 and C_2 is a logical consequence or semi-consequence of C_1 and C_2 .

[Proof]

Let C_1, C_2 and C_3 be denoted as follows:

$$C_1 = X \vee C_1^1, \quad C_2 = \sim X \vee C_2^1, \quad C_3 = C_1^1 \vee C_2^1.$$

The proof will be made in the case (i) $X = F$, (ii) F^* , (iii) T^* , and (iv) T . Suppose there exists an interpretation I for which C_1 and C_2 are T or T^* . In cases (i) and (ii), C_1^1 must be T or T^* , and thus $C_3 = T$ or T^* . In cases (iii) and (iv), C_2^1 must be T or T^* , and thus $C_3 = T$ or T^* . This gives the proof. q.e.d.

Theorem 4

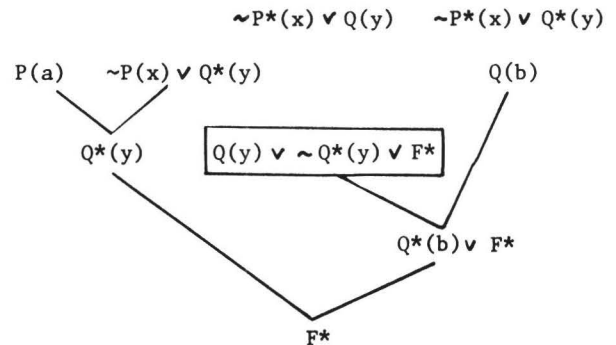
Let S be an unsatisfiable or semi-unsatisfiable set of clauses and $\bar{S}$ be a reduced set of S . If an empty clause is obtained from $\bar{S}$ by applying the resolution principle, then either an empty or semi-empty clause is obtained from S .

[Proof]

Let $\bar{C}_3$ be a resolvent of reduced clauses $\bar{C}_1$ and $\bar{C}_2$. If $\bar{C}_1$ and $\bar{C}_2$ can be resolved, then they must be indicated as follows: $\bar{C}_1 = L \vee C_1^1$, and $\bar{C}_2 = \sim L \vee C_2^1$, where L is a literal. Therefore, we have (i) $C_1 = X \vee C_1^1$, $C_2 = \sim X \vee C_2^1$, or (ii) $C_1 = P \vee C_1^1$, $C_2 = \sim P^* \vee C_2^1$. It is clear that an empty clause is obtained as the resolvent in the case (ii) and that an empty or semi-empty clause is obtained in the case (i).

Fig. 1 illustrates an example of refutation graphs for a set of clauses as follows:

$$S = \{P(a), \quad (P(x) \rightarrow Q(y))^*, \quad \sim Q(b)\} \\ = \{P(a), \quad (\sim P(x) \vee Q(y))^*, \quad \sim Q(b)\} \\ = \{P(a), \quad \sim P(x) \vee Q^*(y), \quad \sim P^*(x) \vee Q(y), \\ \quad \sim P^*(x) \vee Q^*(y), \quad Q(b)\}.$$



$$\sim P^*(x) \vee Q(y) \quad \sim P^*(x) \vee Q^*(y) : \text{unused clauses} \\ Q(y) \vee \sim Q^*(y) \vee F^* : \text{tautology}$$

Fig. 1 A refutation graph for $S = \{P(a), (P(x) \rightarrow Q(y))^*, \sim Q(b)\}$

4. CONCLUDING REMARKS

We have considered a new logic called the ML and derived its mathematical properties. Due to manner operators introduced here, the ML can express more complex statements than the ordinal logic. Since the ML can be considered an extension of the ordinal binary logic, its logical manipulation is quite simple, although it is a four-valued logic. Four values are true(T), true in the sense of "*" (T*), false in the sense of "*" (F*) and false (F). Furthermore, we have developed some interesting applications of the ML to mechanical theorem proving in artificial intelligence. A refutation process for the ML system is similar to that of the ordinal propositional or predicate logic. The idea of semi-unsatisfiability is introduced in the refutation process. That is, a semi-empty clause is obtained as a resolvent if a set of clauses is semi-unsatisfiable. In the refutation process for mechanical theorem proving, therefore, the appearance of a semi-empty clause means that the inference is not completely true but true in the sense of "*". This is one of the advantages of our system.

Several directions seem to be interesting for furthering investigations of the ML from both a mathematical and applicational point of view. The author believes that the ML system is quite useful for solving complex problems.

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