

A MODEL OF DIALOG BASED ON FUZZY SET CONCEPT

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Dialog is considered as the exchange of information between two persons who speak natural language to understand each other. However natural language is so ambiguous both in expression and logic that for understanding it is very important to supplement the information through some reasoning or imagination. In this paper, we analyze this thinking process using fuzzy sets theory and approach a modelling of dialog. The dialog in the model is characterized by the set of objects talked about, the set of their attributes, etc. There are two important factors in the process of dialog: fuzzy information on the objects given by one person and the other person's fuzzy knowledge on the objects. Both fuzzy information and knowledge are expressed by fuzzy sets. The main problem in the paper is to compare these two fuzzy sets with each other. For this purpose the concept of truth qualification in fuzzy logic is useful. The authors propose a generalized truth value instead of an ordinary one which appears in Zadeh's truth qualification rule. Then one of the two persons, say a listener in the dialog, can understand by referring the generalized truth value what the other wants to communicate.

1. INTRODUCTION

In real dialog the roles of a speaker and a listener changes alternately but in our model those are fixed for simplicity. Let us suppose the following situation. The listener acquires a sequence of information from the speaker by repeating indirect questions and guesses what the speaker is thinking. In order to supplement the obtained information by reasoning or imagination, the listener has a priori knowledge on objects which they talk about. Then listener evaluates this information by comparing it with his knowledge and repeats this process until he is convinced that he has understood what the speaker wanted to communicate. Both the information and the knowledge are so ambiguous that fuzzy set concept is suitable for their mathematical expression.

2. PROCESS OF DIALOG

We first describe the situation of dialog between a speaker and a listener by X : a set of objects, A : a set of attributes, AV : sets of values of each attribute. For example $X = \{\text{Yamaguchi, Yoshinaga, Ken, ...}\}$; names of Japanese female singers, $A = \{\text{Age, Appearance, Height, ...}\}$, $AV(\text{Age}) = \{\text{Young, Middle, Old, ...}\}$, $AV(\text{Appearance}) = \{\text{Beautiful, Normal, ...}\}$, etc. Here it is assumed that both the speaker and the listener know the sets X and A . Let x_i be the i -th element of X , a_j the j -th element of A and $av_k(a_j)$ the k -th element of the set $AV(a_j)$

which is the set of values of the attribute a_j . Then dialog proceeds as follows.

The speaker chooses x_i out of the objects.
The listener: How about the Age of x_i ?
The speaker: She is young.
The listener:

Here the listener is forbidden to directly ask the name of x_i and the speaker is allowed to answer only in the form "She is $av_k(a_j)$ ". Generally $av_k(a_j)$ in the form "She is $av_k(a_j)$ " is a fuzzy information on the attribute a_j . Let us assume that this fuzzy information is expressed by a fuzzy set. We further assume that the listener has a priori knowledge on the attributes of the objects, e.g., "She, an object x , is about 20 years old". This type of fuzzy knowledge can be also expressed by a fuzzy set.

3. EVALUATION AND UNDERSTANDING

Let F be a fuzzy set associated with a membership function μ_F which implies a fuzzy information given by the speaker as stated in the previous section. Further let G_i be also a fuzzy set expressing the listener's fuzzy knowledge on an attribute of an object x_i . Now we discuss the process of reasoning in which the listener finds, by using a given information F and a priori knowledges G_i , what the speaker has chosen out of the objects, where F and G_i are of course assumed to have the same universe of discourse concerned with an attribute.

In order to select x_i out of the objects by comparing F with G_i , we can use truth qualification given by Zadeh [1].

$$x_i \text{ is } F \text{ is } \tau_i = x_i \text{ is } G_i$$

where $\tau_i : [0, 1] \rightarrow [0, 1]$ is a linguistic truth value. For example " x_i is young (F) is more or less true (τ_i)" translates into " x_i is about 20 years old (G_i)". If τ_i is false, then the listener can conclude that the speaker is not thinking about the object x_i as far as the attribute under consideration is concerned. Thus the truth value of " x_i is F " with respect to " x_i is G_i " can be used as a criterion for selecting x_i .

According to truth qualification rule as is shown in Fig.1, we have

$$\mu_{G_i}(u) = \mu_{\tau_i}(\mu_F(u)) \quad (1)$$

where u denotes a variable in the universe of discourse of F and G_i .

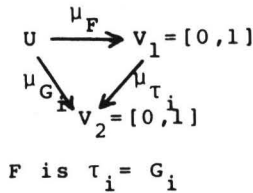


Fig.1 Truth qualification

If μ_F is one to one mapping, then μ_{τ_i} is given by

$$\mu_{\tau_i}(v) = \mu_{G_i}(\mu_F^{-1}(v)), \quad (2)$$

where v denotes a variable in the unit interval $[0, 1]$. In the case that μ_F does not have the inverse, μ_{τ_i} can be obtained, for example, as follows:

$$\mu_{\tau_i}(v) = \sup_{u \in \mu_F^{-1}(v)} \mu_{G_i}(u), \text{ for } \mu_F^{-1}(v) \neq \emptyset$$

$$= 0, \text{ otherwise} \quad (3)$$

That is, $\tau_i = \mu_{G_i}$, the image of G_i through mapping μ_F . In general τ_i is not uniquely determined.

In this paper the authors define instead of $\mu_F(G_i)$ the following relation $R_F(G_i)$ on $[0, 1] \times [0, 1]$.

$$R_F(G_i) = \{(v_1, v_2) \mid v_1 = \mu_F(u), v_2 = \mu_{G_i}(u), u \in U\} \quad (4)$$

The relation $R_F(G_i)$ is considered as a generalization of τ_i . Fig.2 shows the difference between $\mu_F(G_i)$ and $R_F(G_i)$. It is easily seen that the graph of the membership function of a fuzzy set $\mu_F(G_i)$ is a subset of $R_F(G_i)$.

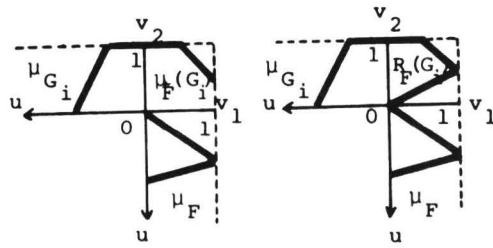


Fig.2 Difference between $\mu_F(G_i)$ and $R_F(G_i)$

Now our problem is how to evaluate $R_F(G_i)$ in comparison of F and G_i . To this end let us define the following values A, B, C and D with respect to $R_F(G_i)$.

$$A = \sup_{(v_1, v_2) \in R_F(G_i)} [1 - v_1 \wedge v_2]$$

$$B = \sup_{(v_1, v_2) \in R_F(G_i)} [1 - v_1 \wedge 1 - v_2] \quad (5)$$

$$C = \sup_{(v_1, v_2) \in R_F(G_i)} [v_1 \wedge 1 - v_2]$$

$$D = \sup_{(v_1, v_2) \in R_F(G_i)} [v_1 \wedge v_2],$$

where A implies the distance in the sense of max-min from a point $(0, 1)$ to its nearest point in $R_F(G_i)$ and B, C, D are those from $(0, 0)$,

$(1, 0), (1, 1)$ respectively as is shown in Fig.3.

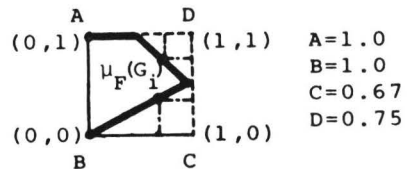


Fig.3 The meaning of A, B, C, D

Next we introduce a kind of order to the class of $R_F(G_i)$'s by a function transforming $R_F(G_i)$

to a real number such that

$$g(R_F(G_i)) = \alpha A' + \beta B + \gamma C' + \delta D, \quad (6)$$

where $A' = 1 - A$, $C' = 1 - C$ and $\alpha, \beta, \gamma, \delta$ are constants.

We have to take into account that this order preserves the partial order concerned with τ_i since $R_F(G_i)$ includes τ_i in its special case.

The partial order $\leq$ of the class of τ_i is defined by $\tau \sqcap \tau' = \tau$ and $\tau \sqcup \tau' = \tau'$ if $\tau \leq \tau'$, where $\sqcap$ is meet* operation and $\sqcup$ is join* operation [2].

* If A and B are fuzzy subsets defined by $A = \sum_i \mu_A(a_i)/a_i$, $B = \sum_j \mu_B(b_j)/b_j$, then

$$A \sqcup B = \sum_{i,j} (\mu_A(a_i) \wedge \mu_B(b_j)) / (a_i \vee b_j)$$

$$A \sqcap B = \sum_{i,j} (\mu_A(a_i) \wedge \mu_B(b_j)) / (a_i \wedge b_j),$$

where $\wedge$ stands for min, $\vee$ for max and Σ for union.

Let us take two fuzzy sets F_1 and F_2 and consider the special case where $R_{F_1}(G_i)$ and $R_{F_2}(G_i)$

are reduced to ordinary functions, more precisely the graphs of membership functions such as μ_{τ_i} in Eq. (2).

Here consider the following two equations.

$$R_{F_1}(G_i) \sqcap R_{F_2}(G_i) = R_{F_1}(G_i) \text{ and} \quad (7)$$

$$R_{F_1}(G_i) \sqcup R_{F_2}(G_i) = R_{F_2}(G_i)$$

$$g(R_{F_1}(G_i)) \wedge g(R_{F_2}(G_i)) = g(R_{F_1}(G_i)) \quad (8)$$

What we should do is to determine α , β , γ and δ in Eq. (6) so that Eq. (7) leads to Eq. (8). The operation in Eq. (7) illustrated in Fig. 4 with the values A' , B , C' and D .

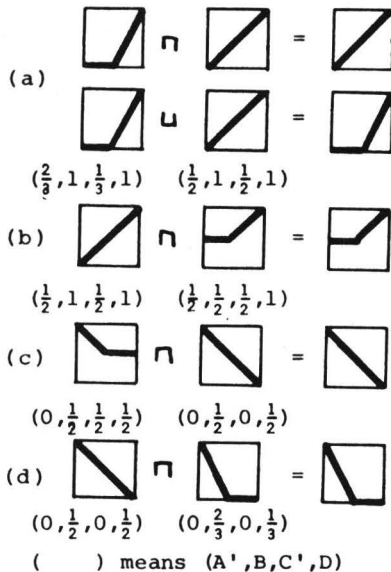


Fig. 4 $\sqcup, \sqcap$ operation

From the above condition we have $\alpha \geq \gamma$ according to Fig. 4(a). From Fig. 4(b), (c) and (d) follow $\beta \geq 0$, $\gamma \geq 0$ and $\delta \geq \beta$. Finally we have the inequalities

$$\delta \geq \beta \geq 0$$

$$\alpha \geq \gamma \geq 0 \quad (9)$$

Conversely, Eq. (8) is obtained from Eq. (7)

using Eq. (9).

Now we can introduce an order to the class of $R_F(G)$'s as follows:

$$R_{F_1}(G) \prec R_{F_2}(G) \Leftrightarrow g(R_{F_1}(G)) < g(R_{F_2}(G)). \quad (10)$$

Let us next consider the general case that the information in the attributes of the object x is sequentially obtained. Let F_j be the information on the j -th attribute and $G_{i,j}$ be a priori knowledge on the j -th attribute of the object x_i . Let us define

$$h(x_i) = \bigwedge_j g(R_{F_j}(G_{i,j})) \quad (11)$$

Here $h(x_i)$ can be interpreted as the degree of the listener's confidence that the speaker is talking about the object x_i . Thus the listener can find, according to the value of $h(x_i)$, an x which the speaker might choose out of the objects.

4. CONCLUSION

A dialog spoken in natural language is the exchange of ambiguous information in many cases. In this paper we develop a mathematical model of a dialog in which one person understands what the partner wants to communicate through ambiguous information and knowledge. In our model of dialog, the behaviour of the listener plays the most important role. If this role is played by computer, then in a restricted sense a man can talk with computer in natural language. The key point of this paper is how to evaluate the given fuzzy information though a priori knowledge. We have presented a generalized truth value and introduced an order into its class for the evaluation.

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- [2] M. Mizumoto and K. Tanaka, "Some Properties of Fuzzy Sets of Type 2", Inform. & Control, 31 (1976) 312-340.