

INFERRING THE DIRECTION OF THE SUN FROM INTENSITY VALUES ON A GENERALISED CONE

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An initial attempt to synthesise some of the ideas of Marr and Horn about surface geometry and lighting is described. In some situations the direction of the sun can be deduced from intensity values on a section of a generalised cone which flanks a bounding contour.

1. INTRODUCTION

A key problem in developing a computational theory of visual perception is to discover the constraints which hold between lighting, and the geometry and photometric characteristics of a surface, which together with the prior assumptions of a viewer enable the interpretation of a two dimensional image as depicting a three-dimensional scene. The work reported here arose from a study of the ever-present problem of determining occlusion relationships in a single image. A companion paper [1] argues firstly that the current stock of ideas about occlusion are extremely limited, and secondly that the best way forward currently seems to be to aim for a synthesis of the work of Marr [3] and Horn [2]. Marr [3] argues persuasively that a viewer has strong preconceptions regarding the three dimensional surface which generated a given silhouette in an image, and proves that under a number of precisely stated assumptions these preconceptions amount to requiring that the generating surface be a "generalised cone". Several accounts of generalised cones have appeared in print, and we adopt here the restricted version defined by Marr [3]. A generalised cone results from moving a twice continuously differentiable planar closed curve $\rho(r, \theta)$, called the cross-section function along a straight axis Λ , subject to expansion according to a twice continuously differentiable positive real-valued function $h(z)$, [2, p.467 and fig. 5]. Horn [2] shows how surface geometry, lighting and the reflectance characteristics of a surface combine to produce intensity values, and points out that in some cases one can deduce surface geometry from intensity shading. Woodham [5] and Ullman (personal communication) have pointed out that although one can make

surprisingly detailed assertions about the curvature of a surface from a small region in an image, it is hard to decide whether it is convex or concave. However, this ambiguity is usually resolved if one can see a portion of the bounding contour of an object. This suggests that a good way to begin synthesising the ideas of Marr and Horn is to see what can be deduced when the contour region of a generalised cone is illuminated. This paper describes an initial attempt to develop such an analysis, and shows that in some cases the direction of the sun in an image can be deduced from the intensities near the surface of a generalised cone.

2. THE BASIC APPROACH

Following Marr [2, p.467], "let V be a distant vantage point for the generalised cone Σ such that (i) the image formed from V is an orthogonal projection, and (ii) the rays in the projection all be parallel to the plane of the cross-section of Σ ". V is associated with a specific angle ϕ in the polar coordinate system of ρ . Let $\underline{v}$ be a unit vector in the direction of V . The contour generator Γ_θ is the set of points

$\{(\rho(\theta) \cdot h(z), \theta, z) | z\}$ and lies in the same section plane Ω_θ as Λ . The normal $\underline{n}$ to the surface Σ at a point on Γ_θ can be separated into two components, namely its projection $\underline{n}_\theta$ onto the section Ω_θ , and its projection $\underline{n}_\rho$ onto the cross-section ρ . Since one can decouple the expansion function h and cross-section function ρ for a generalised cone $\Sigma = h \times \rho$, differentiating along Γ_θ leaves $\underline{n}_\rho$ constant, so that $\Delta \underline{n}$ is equal to $\Delta \underline{n}_\theta$. We shall be interested in

pixels close to the specific contour generator Γ_ϕ , and it is reasonable to suppose for a sufficiently fine spatial image resolution that they lie on a contour generator Γ_θ where θ is close to ϕ .

Let an arbitrary vector direction $\underline{l}$ be given. Later $\underline{l}$ will be specialised to point towards the single distant light source (the sun). Suppose we denote the scalar product $\underline{l} \cdot \underline{n}$ by X . Consider a short displacement along Γ_ϕ , say Δs , so that the surface normal changes from $\underline{n}_1$ to $\underline{n}_2$.

$$\begin{aligned}\Delta X &= \underline{l} \cdot \underline{n}_1 - \underline{l} \cdot \underline{n}_2 \\ &= \underline{l} \cdot \Delta \underline{n} \\ &= \underline{l} \cdot \Delta \underline{n}_\phi\end{aligned}$$

since we are differentiating along Γ_ϕ . If we consider a Horn-like model of illumination, X can be the intensity $\text{Int} = \phi(I, E, G)$ [3]. In this way, we hope to extract $\underline{l}$, the sun position, from $\Delta \text{Int} / \Delta s$.

3. COORDINATES

In order to flesh out the above proposal, we need to be aware that there is a problem regarding the coordinate systems we use. The "natural" coordinate frame for a generalised cone is one that is local to it: cylindrical coordinates z along the axis, and the girdle angle ϕ about the axis Λ relative to some dorsal zero. The "natural" coordinate frame for a vector field, to facilitate the computation of scalar products, is a global frame, such as a Cartesian system. In fact, Marr and Nishihara [4] faced precisely this difficulty and their methods propose a solution in the form of specifying a space arrow vector ("spasar") relative to a given axis ("axis"). In fact, their methods apply to specify a general vector relative to the axis Λ of the generalised cone Σ . Given an origin on Λ , they suggest encoding a vector of magnitude s at a point Q on the surface of Σ using six parameters. The first three parameters (z, r, θ) specify the point Q , while the other three (α, β, s) define the vector. Here α is the inclination of the vector to Λ and β the girdle angle of the vector about Λ .

In the cases considered here, Q will always be the position on Γ_ϕ at which the normal is computed, and it will be suppressed. Moreover, we are only interested in vector directions and so we dispense with the norm s . Thus two parameters (α, β) are required to specify a vector. We need

to compute the scalar product of two such vectors, and the following easily proved lemma achieves this.

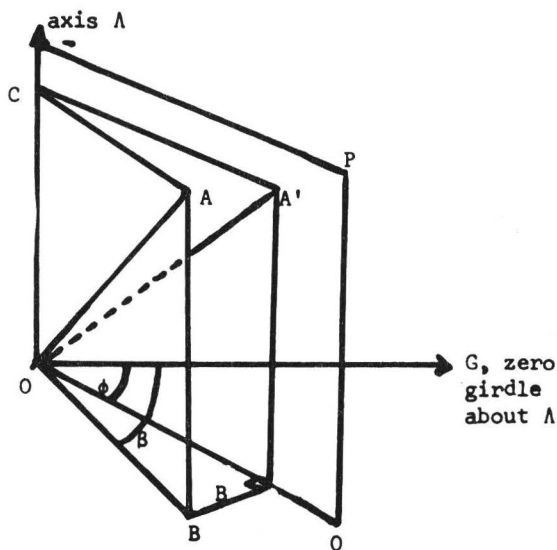
Lemma 1 The scalar product of (α_1, β_1) and (α_2, β_2) is given by $\cos \alpha_1 \cos \alpha_2 + \sin \alpha_1 \sin \alpha_2 \cos(\beta_1 - \beta_2)$.

Suppose we have a vector (α, β) . We shall need to consider its projection onto the section plane Ω_ϕ containing the axis Λ and a vector with girdle angle ϕ .

Lemma 2 The projection on (α, β) on Ω_ϕ has girdle angle ϕ and inclination μ where

$$\tan \mu = \tan \alpha \cos(\beta - \phi)$$

Proof: (refer to Fig. 1).



$$\hat{B}OG = \beta, \quad \hat{A}OC = \alpha, \quad \hat{Q}OG = \phi.$$

The projection of $\vec{OA}$ onto the plane OPQ is defined by keeping $OC = |OA| \cos \alpha$ and projecting AB onto $A'B'$ in PQ . Now $|OB'| = |OB| \cos(\beta - \phi) = |OA| \sin \alpha \cos(\beta - \phi)$. Thus the inclination of $\vec{OA'}$ is $\arctan |OB'| / OC = \tan \alpha \cos(\beta - \phi)$.

Fig. 1

4. THE NORMALS OF A GENERALISED CONE AND THE MAIN RESULT

Consider the section Ω_ϕ of a generalised cone

(Fig. 2). The contour generator Γ_ϕ is $\{(z, h(z) * \rho(\phi) * \sin\psi_\phi) | z \text{ on axis}\}$ where ψ_ϕ is the eccentricity of the cross radial ρ in the section Ω_ϕ . Clearly, the direction of the tangent is given by

$$\tan\alpha = h'(z) * \rho(\phi) * \sin\psi_\phi$$

so the tangent vector is (α, ϕ) . The normal $\underline{n}_\phi$ is given by $(90 - \alpha, \phi)$.

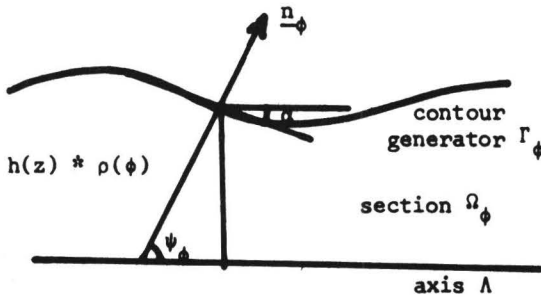


Fig. 4

Consider now an arbitrary vector (λ, μ) and define the scalar product $X = (\lambda, \mu) \cdot (90 - \alpha, \phi)$. At a neighbouring point of the contour generator Γ_ϕ , $\alpha' = \alpha + \Delta\alpha$. As in section 2

$$\frac{\Delta X}{\Delta s} = \frac{1}{\Delta s} \{ (90 - \alpha, \phi) \cdot (\lambda, \mu) - (90 - \alpha', \phi) \cdot (\lambda, \mu) \}$$

where s is taken along Γ_ϕ . By lemma 1,

$$\begin{aligned} \Delta X / \Delta s &= \frac{1}{\Delta s} \{ \cos(90 - \alpha) \cos\lambda \\ &+ \sin(90 - \alpha) \sin\lambda \cos(\phi - \mu) \\ &- \cos(90 - \alpha') \cos\lambda - \\ &- \sin(90 - \alpha') \sin\lambda \cos(\phi - \mu) \} \\ &= \frac{1}{\Delta s} \{ \cos\lambda (\sin\alpha - \sin\alpha') \\ &- \sin\lambda \cos(\phi - \mu) (\cos\alpha - \cos\alpha') \} \end{aligned}$$

recall that

$$\sin\alpha' \approx \sin\alpha + \cos\alpha \Delta\alpha$$

and $\cos\alpha' \approx \cos\alpha - \sin\alpha \Delta\alpha$. Thus

$$\frac{\Delta X}{\Delta s} = \frac{\Delta\alpha}{\Delta s} (\cos\lambda \cos\alpha - \sin\lambda \sin\alpha \cos(\phi - \mu)).$$

Now $\Delta\alpha/\Delta s$ is the curvature of Γ_ϕ and can be measured in the silhouette. We denote it by

$\text{curvat}_\phi(z)$. Thus

$$\frac{\Delta X}{\Delta s} = \text{curvat}_\phi(z) * (\cos\lambda \cos\alpha - \sin\lambda \sin\alpha \cos(\phi - \mu)) \quad (1)$$

We now specialise the vector (λ, μ) used in the above derivation to be the sun vector (λ_s, μ_s) .

Clearly, there is no change in the phase angle G along Γ_ϕ and so following Horn [2] we have

$$\begin{aligned} \frac{\partial \text{Intensity}}{\partial s} &= \frac{\partial \phi(I, E)}{\partial s} \\ &= \frac{\partial \phi(I, E)}{\partial I} \cdot \frac{\partial I}{\partial s} + \frac{\partial \phi(I, E)}{\partial E} \cdot \frac{\partial E}{\partial s}. \end{aligned}$$

We can now apply formula (1) deduced above to compute $\frac{\partial I}{\partial s}$ using the sun vector (λ_s, μ_s) and $\partial E/\partial s$ using the view vector $\underline{v}$. This doesn't exploit the fact that the contour Γ_ϕ along which we are differentiating is a bounding contour, however.

There are two ways to incorporate this extra constraint.

1. Take the general formula for the surface normal $\underline{n}$ and use the fact that $\underline{n} \cdot \underline{v}$ is zero, since $\underline{n}$ is orthogonal to $\underline{v}$. This has the disadvantage that the formula defining $\underline{n}$ is rather complex. It can be cleaned up by restricting $\underline{v}$ to line in a plane parallel to ρ as in Marr's [3] first three theorems; but this misses the essence of the idea that it is largely irrelevant as $\Delta \underline{n} = \Delta \underline{n}_\phi$ along Γ_ϕ .

2. Build the assumption $e = \pi/2$ (i.e., $E = \text{zero}$) directly into the function ϕ , so that ϕ is a function only of I, G . We then get $\phi(I, \emptyset, G) = R(I, G)$, so that

$$\begin{aligned} \frac{\partial \text{Intensity}}{\partial s} &= \frac{\partial R}{\partial I} * \text{curvat}_\phi(z) * \\ &\quad [\cos\alpha \cos\lambda_s - \sin\alpha \sin\lambda_s \cos(\phi - \mu)] \end{aligned}$$

In particular, at a stationary point of intensity along Γ_ϕ , $\partial \text{Intensity}/\partial s$ is zero. If we assume that neither $\text{curvat}_\phi(z)$ nor $\partial R/\partial I$ are zero, we get

$$\cos\alpha \cos\lambda_s = \sin\alpha \sin\lambda_s \cos(\phi - \mu_s)$$

so that $\cot\alpha = \tan\lambda_s \cos(\phi - \mu_s)$

which by lemma 2 is the projection of (λ_s, μ_s) onto the sector containing (α, ϕ) , that is, the image plane.

Hence the direction of the sun can be deduced from a stationary point of intensities taken along a section which flanks a bounding contour. Future work will develop the idea that the assumption of a Marr generalised cone allows one to make useful inferences from shading.

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