# Multi-View Learning with Limited and Noisy Tagging

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## Abstract

Multi-view tagging has become increasingly popular in the applications where data representations by multiple views exist. A robust multi-view tagging method must have the capability to meet the two challenging requirements: limited labeled training samples and noisy labeled training samples. In this paper, we investigate this challenging problem of learning with limited and noisy tagging and propose a discriminative model, called MSMC, that exploits both labeled and unlabeled data through a semi-parametric regularization and takes advantage of the multi-label space consistency into the optimization. While MSMC is a general method for learning with multi-view, limited, and noisy tagging, in the evaluations we focus on the specific application of noisy image tagging with limited labeled training samples on a benchmark dataset. Extensive evaluations in comparison with state-ofthe-art literature demonstrate that MSMC outstands with a superior performance.

# 1 Introduction

Multi-view tagging has become increasingly popular in the applications where data representations by multiple views exist. It aims at improving generalization performance by learning tagging tasks from multiple views simultaneously. Multi-view tagging has shown a strong power in helping develop effective solutions to many real-world problems. Most of the existing multi-view tagging algorithms are proposed by imposing a similarity constraint between two distinct single view taggings. Generally, two requirements are imposed for a robust multi-view learning method. First, since it is very timeconsuming and labor-intensive for manually labeling data, it is expected that a robust multi-view tagging method needs only a small portion of the labeled training samples. Second, even with the manual labeling of images, there is no guarantee that the provided labels are always correct. In many practical engineering applications, the obtained training data are often contaminated by noise. Figure 1 shows exemplar images with noisy tagging. In Figure 1(a), the given tags of the image are airplane and sky, while it is obvious that the tag airplane is given incorrectly. In Figure 1(b), the given tags of the image



Figure 1: Exemlpar images with noisy tagging.



Figure 2: Exemplar images with multiple tags.

are *person*, *sea*, and *bear*, while it is obvious that the tag *bear* is given incorrectly.

This paper is motivated to address both requirements for developing a robust multi-view tagging method. First, we want to make use of the unlabeled data through a semiparametric regularization. In particular, we consider the case where the unlabeled data are expected to determine the marginal distribution of the data if there is a small set of labeled data available along with a relatively large set of unlabeled data. Thus, we must consider the geometric structure of the marginal distribution of the whole data including the labeled data and the unlabeled data. Moreover, semi-parametric regularization are applied to explore the geometric structure of the whole data.

Second, we want to mitigate the influence of noisy tags by exploiting the whole information contained in the tag space. Figure 2 shows exemplar images with multiple tags. The images in the first row in Figure 2, which are all tagged as *bird*, always have the accompanied tags of *sky*, *cloud*, and *tree*, while the images in the second row in Figure 2, which are

all tagged as *fish*, always have the accompanied tags of *wa*ter, coral, and ocean. Obviously, these accompanied tags can be utilized as an additional feature to help better distinguish images tagged as *bird* from images tagged as *fish*. However, in the one-vs-all (OVA) mode, most of the Support Vector Machine (SVM)-based methods only utilize one tag of the data at a time, and ignore the other tags the data contain at the same time. For multi-view learning, since the multi-label space, which is the constitution of accompanied tags, can be considered as an additional shared feature between different views, we make use of it to build multi-label space consistency between samples from different views to mitigate the influence of the noise.

In this paper we investigate the multi-view tagging problem with a new perspective of considering semi-parametric regularization and multi-label space consistency simultaneously. In particular, we present Multi-view Semi-parametric Support Vector Machine with Multi-label Space Consistency (referred as MSMC), a discriminative method for multi-view tagging. The key idea lies in incorporating the geometric structure of the unlabeled data with the semi-parametric regularization and extending the similarity constraints between predictions on the samples from multiple views with the multi-label space consistency constraint.

While MSMC is a general method, we demonstrate through extensive evaluations in the application of multi-view image tagging using real data that the proposed method performs well in comparison with the peer methods in the literature as an effective and promising solution to the problem of semi-supervised multi-view learning with limited and noisy image tagging.

#### 2 Related work

We begin by reviewing the literature on semi-supervised learning and multi-view learning, and then review the related work on the tagging methods. Finally, we introduce the literature in the field of noisy learning.

Most existing semi-supervised learning methods have been casted as a regularization problem in the literature. As a representative method, transductive SVM [Vapnik, 1998; Xu *et al.*, 2007] is considered as combining fully supervised SVM with an additional regularization term on the unlabeled data. Semi-parametric regularization is also an attractive solution to semi-supervised problems [Smola *et al.*, 1998; Ruppert *et al.*, 2003; Guo *et al.*, 2008; Bouboulis *et al.*, 2010; Li *et al.*, 2013].

Multi-view learning utilizes the consensus among learners trained on different views to improve the overall classification result [Xu *et al.*, 2013]. It has been extensively used in the applications where data representations by multiple views exist [Sindhwani and Rosenberg, 2008; Rosenberg *et al.*, 2009; Li *et al.*, 2010; Liu *et al.*, 2015]. Farquhar et al. [Farquhar *et al.*, 2006] proposes SVM-2K by imposing a similarity constraint between two distinct SVMs each trained from one view of the data.

Tagging methods in the literature are mainly categorized into two types: generative methods and discriminative methods [Blei and Jordan, 2003; Zha *et al.*, 2008; Zhang and Zhou, 2014]. Most generative methods introduce a set of latent variables to learn the joint distribution of the image features and semantic labels [Barnard *et al.*, 2003]. Discriminative methods reduce the multi-label problem to a set of binary classification problems. The representative techniques for this category of approaches are extensions of SVM, which have demonstrated a strong discriminative power [Goh *et al.*, 2001; Qi and Han, 2007; Yang *et al.*, 2006].

Noisy tagging refers to scenarios of attribute noise and class noise [Zhu and Wu, 2004]. In this paper, we concentrate on the problem raised by class noise. There are a number of denoising methods for classification; they can be further classified into two categories: filtered preprocessing of the data and robust design of the algorithms. In the former category, filtered preprocessing is developed to remove the noise from the training set as much as possible [Van Hulse and Khoshgoftaar, 2006; Zhu et al., 2003]. For the latter category, robust algorithms are designed to reduce the impact of the noise in the classification [Lin and de Wang, 2004; Liu and Zheng, 2007; Tang et al., 2011; Qi et al., 2012]. Qi et al. [Qi et al., 2012] proposes a multi-view noisy tagging method, called MSS-2K, which exploits the information from a multi-label space. Further, they introduce an active learning scheme called MITL to improve the performance of MSS-2K. In addition, tag refinement is considered as the auxiliary work for image noisy tagging in the literature [Wang *et al.*, 2007; Zhu et al., 2010; Liu et al., 2010].

# **3** Semi-parametric Regularization Learning

#### Semi-parametric Regression

In statistics, semi-parametric regression refers to regression models that combine parametric and nonparametric models.

For example, suppose that we want to estimate an unknown function from a set of labeled data points  $(\mathbf{x}_i, y_i)$ ,  $1 \le i \le l$  by minimizing

$$\bar{f}^* = \arg\min_{\bar{f}} \frac{1}{l} \sum_{i=1}^{l} L(\mathbf{x}_i, y_i, \bar{f}(\mathbf{x}_i))$$
(1)

where  $L(\cdot)$  is a loss function.

Then for parametric regression models,  $\overline{f}$  can be given as an explicit function which is dependent on a finite number of parameters (e.g., linear regression); for non-parametric regression models,  $\overline{f}(\mathbf{x})$  cannot be estimated via an explicit function; for semi-parametric regression models,  $\overline{f}$  can be decomposed into two parts as  $\overline{f} \triangleq f + h$ , where f is a nonparametric part which can be estimated from the data set,  $h \in \text{span}\{\psi_p\}_{p=1}^M$ , where  $\{\psi_p\}_{p=1}^M : \mathcal{X} \to \mathbb{R}$  is a family of parametric functions.

Minimizing the framework in Eq. (1) may lead to numerical instabilities and a bad generalization performance [Schölkopf and Smola, 2001]. A possible solution is to add stabilization (regularization) term to the above minimization problem. This leads to a better conditioning of the problem. Thus, we consider the following minimization problem

$$\bar{f}^* = \arg\min_{\bar{f}} \frac{1}{l} \sum_{i=1}^{l} L(\mathbf{x}_i, y_i, \bar{f}(\mathbf{x}_i)) + \gamma_1 \Omega_1[f] + \gamma_2 \Omega_2[h]$$
(2)

where  $\gamma_1 > 0$  and  $\gamma_2 > 0$  are regularization parameters. When we equivalently think the feature space as a reproducing kernel Hilbert space,  $\Omega_1[f]$  is the norm of the RKHS  $\mathcal{H}_K$  representation of the feature space  $||f||_K^2$ ,  $\Omega_2[h]$  is the norm of the parametric space  $||h||_{\Psi}^2$ , where  $|| \cdot ||_{\Psi}$  is the norm in  $\Psi \triangleq \operatorname{span}\{\psi_p\}_{p=1}^M$ . In this case, we equivalently minimize

$$\bar{f}^* = \arg\min_{\bar{f}} \frac{1}{l} \sum_{i=1}^{l} L(\mathbf{x}_i, y_i, \bar{f}(\mathbf{x}_i)) + \gamma_1 \|f\|_K^2 + \gamma_2 \|h\|_{\Psi}^2$$
(3)

where  $\bar{f} \triangleq f + h$  with  $f \in \mathcal{H}_K$ , and  $h \in \Psi$ .

The following semi-parametric Representer Theorem states that the solution to the minimization problem in Eq. (3) exists in  $\mathcal{H}_K$  and  $\Psi$ , and gives the explicit form of a minimizer.

**Theorem 1.** (Semi-parametric Representer Theorem [Schölkopf and Smola, 2001]): Denote by  $\Omega: [0, \infty) \times [0, \infty) \to \mathbb{R}$  a strictly monotonically increasing function, by  $\mathcal{X}$  a set, and by  $c: (\mathcal{X} \times \mathbb{R}^2)^l \to \mathbb{R} \cup \{\infty\}$  an arbitrary loss function. Let  $\{\psi_p\}_{p=1}^M: \mathcal{X} \to \mathbb{R}$  be a set of Mreal valued functions with the property that the  $l \times M$  matrix  $(\psi_p(\mathbf{x}_i))_{ip}$  has rank M and  $\text{span}\{\psi_p\} \triangleq \Psi \subset \mathcal{H}_K^\perp$  has the norm  $\|\cdot\|_{\Psi}$ . Then for any  $\bar{f} \triangleq f + h$  with  $f \in \mathcal{H}_K$  and  $h \in \Psi$ , minimizing the regularized risk

$$c((\mathbf{x}_1, y_1, f(\mathbf{x}_1)), \dots, (\mathbf{x}_l, y_l, f(\mathbf{x}_l))) + \Omega(||f||_K, ||h||_{\Psi})$$

admits a representation of the form

$$\bar{f}(\mathbf{x}) = \sum_{i=1}^{l} \alpha_i K(\mathbf{x}_i, \mathbf{x}) + \sum_{p=1}^{M} \beta_p \psi_p(\mathbf{x})$$
(4)

with  $\alpha_i, \beta_p \in \mathbb{R}$ .

#### **Learning Parametric Functions**

Let  $\mathbf{x}_1, \ldots, \mathbf{x}_{l+v} \in \mathbb{R}^d$  denote a set of inputs including l labeled data points and v unlabeled data points. For semiparametric regression, we might hope that the geometric structure of the whole data distribution can be exploited for a better function learning. There are suitable choices for the parametric functions. Specifically, we adopt the same strategy as that in [Guo *et al.*, 2008] to obtain the parametric functions by applying Kernel Principal Component Analysis (KPCA) algorithm [Schölkopf *et al.*, 1997] to the whole data set. KPCA finds the principal axes in the feature space which carry more variance than any other directions by diagonalizing the covariance matrix.

$$\Sigma = \frac{1}{l+\upsilon} \sum_{j=1}^{l+\upsilon} \phi(\mathbf{x}_j) \phi(\mathbf{x}_j)^T$$
(5)

where  $\phi$  is a mapping function in the RKHS.

To find the principal component, we solve the eigenvalue problem,  $\Sigma \mathbf{W} = \Lambda \mathbf{W}$ . Let  $\Lambda$  denote the M largest eigenvalues and  $\mathbf{W}$  the corresponding eigenvector space. Given the data point  $\mathbf{x}$ , the projection onto the principal axes is given by  $\phi(\mathbf{x})^T \mathbf{W}$ . Therefore, we can set the parametric function for  $\phi(\mathbf{x})$ :

$$\psi(\mathbf{x}) = \phi(\mathbf{x})^T \mathbf{W} \tag{6}$$

Since W is a linear combination of the kernel functions in RKHS, the geometric structure of the marginal distribution of the data can be obtained by this learned parametric function. Further, the geometric structure of the data distribution is incorporated by a semi-parametric regularization. In this way, we obtain the minimizer of Eq. (3) as follows:

$$\bar{f}^*(\mathbf{x}) = \sum_{i=1}^l \alpha_i^* K(\mathbf{x}_i, \mathbf{x}) + \sum_{p=1}^M \beta_p^* \psi_p(\mathbf{x})$$
(7)

where K is the kernel in the original RKHS  $\mathcal{H}_K$ .

#### 3.1 Semi-parametric SVM

Based on the above derivation, we extend the standard SVM to semi-parametric SVM by solving the following optimization problem:

$$\min_{\mathbf{w},\beta} \frac{1}{2} \|\mathbf{w}\|^2 + \frac{1}{2} \sum_{p=1}^M \beta_p^2 + C \sum_{i=1}^l \xi_i$$
s.t.  $y_i(\langle \mathbf{w}, \phi(\mathbf{x}_i) \rangle + \langle \beta, \psi(\mathbf{x}_i) \rangle) \ge 1 - \xi_i$   
 $\xi_i \ge 0 \ i = 1, \dots, l$ 
(8)

where  $\beta = (\beta_1, \dots, \beta_{M+1})$  and  $\psi(\mathbf{x}_i) = (1, \psi_1(\mathbf{x}_i), \dots, \psi_M(\mathbf{x}_i))$ . In particular, the semi-parametric SVM reduces to the standard SVM when M = 0.

# 4 Multi-view Semi-parametric SVM with Multi-label Space Consistency

#### 4.1 Notations for Multi-label Space

In this paper we concentrate on the two-view case. It is easy to extend the proposed method to more than two views. Two views of the dataset  $\mathcal{I}$  are denoted as  $\mathcal{V}^{(a)}$  and  $\mathcal{V}^{(b)}$ , respectively. Each instance  $I_i \in \mathcal{I}$  is labeled with various tags. The whole tag vocabulary for  $\mathcal{I}$  forms the *E*-dimensional multilabel space  $\mathcal{T}$ . When one tag  $T_r$   $(1 \leq r \leq E)$  is chosen as the classification target, the other tags can form the additional feature space of tags, denoted as  $\mathcal{L}_r$ . Obviously, the dimensionality of  $\mathcal{L}_r$  is E - 1. Let an *E*-dimensional vector  $\mathbf{d}_i = (d_{i,1}, d_{i,2}, \ldots, d_{i,E})'$  be the tag representation for  $I_i$ , where  $d_{i,r} \in \{0,1\}, 1 \leq r \leq E$  represents the occurrence of the  $r^{th}$  tag  $T_r$  for  $I_i$ . For each  $I_i$  and each  $T_r$ , we denote  $y_{i,r}$  as the class label of  $I_i$ , where  $y_{i,r} = 2 \cdot d_{i,r} - 1$ .  $I_i = (\mathbf{x}_i^{(a)}, \mathbf{x}_i^{(b)}, \mathbf{d}_i)$ , where  $\mathbf{x}_i^{(a)}$  and  $\mathbf{x}_i^{(b)}$  are the feature descriptors of  $I_i$ .

## 4.2 Motivation of MSMC

In the two views learning, we assume that the labeled data are of the following formulation:  $I_i = {\mathbf{x}_i^{(a)}, \mathbf{x}_i^{(b)}, y_i}_{i=1}^n$ , where  $\mathbf{x}_i^{(a)}$  ( $\mathbf{x}_i^{(b)}$ ) is a feature vector of  $I_i$  in view  $\mathcal{V}^{(a)}$  ( $\mathcal{V}^{(b)}$ ), and  $y_i \in {-1, +1}$  is a class label of  $I_i$ . The classical SVM-2K imposes a similarity constraint between two distinct SVMs each trained from one view of the data. The constraint they introduce into the optimization is

$$|f^{(a)}(\mathbf{x}_{i}^{(a)}) - f^{(b)}(\mathbf{x}_{i}^{(b)})| \le \eta_{i}, \qquad \eta_{i} \ge 0$$
(9)

where  $f^{(a)\setminus(b)}(\cdot)$  are the SVM decision functions belonging to each of the two views by superscripts a and b, respectively,  $\eta_i$  is a variable that improves the consensus between the two views.

A first approach to multi-view semi-parametric learning is to combine the above constraint with the semi-parametric SVM objective function for each view,

$$|\bar{f}^{(a)}(\mathbf{x}_i^{(a)}) - \bar{f}^{(b)}(\mathbf{x}_i^{(b)})| \le \eta_i^{(ab)}, \qquad \eta_i^{(ab)} \ge 0 \qquad (10)$$

where  $\bar{f}^{(a)\setminus(b)}(\cdot)$  are the semi-parametric SVM decision functions belonging to each of the two views by superscripts a and b, respectively.

However, this model suffers from the limitation that it cannot exploit the information contained in the tag space. Assume that we have three images, where the first has tags motor, people, and road, the second has tags people and road, and the third has tags people, TV, and office. When the tag people is chosen as the classification target, the second image is more similar to the first one than the third one because it shares *road* with the first image while the third image has no road. From this point of view, the traditional multi-view learning methods that ignore the tag information may not be appropriate.

Specifically, in the OVA mode, when one tag  $T_r$  is chosen as the classification target, the other tags can form the additional feature space of the tags  $\mathcal{L}_r$ . We denote the feature vector of  $I_i$  in  $\mathcal{L}_r$  as  $\mathbf{t}_{i,r}$ , where  $\mathbf{t}_{i,r} =$  $(d_{i,1},\ldots,d_{i,r-1},d_{i,r+1},\ldots,d_{i,E})'$ . The neighborhood of  $I_i$ in  $\mathcal{L}_r$  (including  $I_i$  itself) is denoted as  $\mathcal{N}_r(I_i)$ . The neighborhood information can then be added into the learning between the views by introducing the following consistency constraints:

$$\begin{aligned} \forall_{i=1}^{l} \ and \ \forall_{j} \in \mathcal{N}_{i} : \\ |\bar{f}^{(a)}(\mathbf{x}_{i}^{(a)}) - \bar{f}^{(b)}(\mathbf{x}_{i}^{(b)})| \le \eta_{ij}^{(ab)}, \ \eta_{ij}^{(ab)} \ge 0 \end{aligned} \tag{11}$$

$$|\bar{f}^{(b)}(\mathbf{x}_{i}^{(b)}) - \bar{f}^{(a)}(\mathbf{x}_{j}^{(a)})| \le \eta_{ij}^{(ba)}, \ \eta_{ij}^{(ba)} \ge 0$$
(12)

where  $\mathcal{N}_i \triangleq \{j | I_i \in \mathcal{N}_r(I_i)\}$ . We show that constraint (12) can be approximately obtained by constraint (11). For  $i \in \mathcal{N}_i$ , constraint (11) works. For  $i \notin \mathcal{N}_j$ , we obtain the following constraint based on constraint (11):

$$\begin{aligned} |\bar{f}^{(b)}(\mathbf{x}_{i}^{(b)}) - \bar{f}^{(a)}(\mathbf{x}_{j}^{(a)})| \\ &= |\bar{f}^{(b)}(\mathbf{x}_{i}^{(b)}) - \bar{f}^{(a)}(\mathbf{x}_{i}^{(a)}) + \bar{f}^{(a)}(\mathbf{x}_{i}^{(a)}) - \bar{f}^{(b)}(\mathbf{x}_{j}^{(b)}) \\ &+ \bar{f}^{(b)}(\mathbf{x}_{j}^{(b)}) - \bar{f}^{(a)}(\mathbf{x}_{j}^{(a)})| \\ &\leq |\bar{f}^{(b)}(\mathbf{x}_{i}^{(b)}) - \bar{f}^{(a)}(\mathbf{x}_{i}^{(a)})| + |\bar{f}^{(a)}(\mathbf{x}_{i}^{(a)}) - \bar{f}^{(b)}(\mathbf{x}_{j}^{(b)})| \\ &+ |\bar{f}^{(b)}(\mathbf{x}_{j}^{(b)}) - \bar{f}^{(a)}(\mathbf{x}_{j}^{(a)})| \\ &\leq \eta_{ii}^{(ab)} + \eta_{ij}^{(ab)} + \eta_{jj}^{(ab)} \triangleq \bar{\eta}_{ij} \end{aligned}$$
(13)

From the above inference, constraint (12) coincides with constraint (11) approximately with a little larger constraint variable. Thus, we only select constraint (11) in order to reduce the computational complexity.

### 4.3 Formulation of MSMC

Combining the two-view constraint (11) with the semiparametric SVM and allowing different regularization constants, we obtain the following optimization for multi-view semi-parametric SVM with multi-label space consistency (MSMC):

$$\begin{split} \min_{\mathbf{w},\beta} \ \frac{1}{2} \|\mathbf{w}^{(a)}\|^2 &+ \frac{1}{2} \|\mathbf{w}^{(b)}\|^2 + \frac{1}{2} \|\beta^{(a)}\|^2 + \frac{1}{2} \|\beta^{(b)}\|^2 \\ &+ C^{(a)} \sum_{i=1}^l \xi_i^{(a)} + C^{(b)} \sum_{i=1}^l \xi_i^{(b)} + \sum_{i=1}^l \sum_{j \in \mathcal{N}_i} C^{(ab)}_{ij} \eta^{(ab)}_{ij} \\ &C^{(ab)}_{ij} = \left\{ \begin{array}{c} C' \\ C^* / e^{Sim(\mathbf{t}_{i,r}, \mathbf{t}_{j,r})} & i \neq j \end{array} (C^* < C') \\ &(14) \end{array} \right. \\ &s.t. \quad \forall_{i=1}^l : \\ y_i \Big( \langle \mathbf{w}^{(a)}, \phi(\mathbf{x}_i^{(a)}) \rangle + \langle \beta^{(a)}, \psi(\mathbf{x}_i^{(a)}) \rangle \Big) \ge 1 - \xi_i^{(a)}, \xi_i^{(a)} \ge 0 \\ y_i \Big( \langle \mathbf{w}^{(b)}, \phi(\mathbf{x}_i^{(b)}) \rangle + \langle \beta^{(b)}, \psi(\mathbf{x}_i^{(b)}) \rangle \Big) \ge 1 - \xi_i^{(b)}, \xi_i^{(b)} \ge 0 \\ &\forall_{i=1}^l \ and \ \forall j \in \mathcal{N}_i : \\ \left| \langle \mathbf{w}^{(a)}, \phi(\mathbf{x}_i^{(a)}) \rangle + \langle \beta^{(a)}, \psi(\mathbf{x}_i^{(a)}) \rangle - \langle \mathbf{w}^{(b)}, \phi(\mathbf{x}_j^{(b)}) \rangle \\ &- \langle \beta^{(b)}, \psi(\mathbf{x}_i^{(b)}) \rangle \Big| \le \eta_{ij}^{(ab)}, \ \eta_{ij}^{(ab)} \ge 0 \end{split}$$

where  $Sim(\mathbf{t}_{i,r}, \mathbf{t}_{j,r})$  represents the Jaccard similarity coefficient between  $I_i$  and  $I_j$  in  $\mathcal{L}_r$ .

Based on the above analyses, the algorithm of MSMC is outlined in Algorithm 1.

Algorithm 1: MSMC Algorithm								
	<b>Input</b> : Datasets $X_L^{(a)}$ , $X_L^{(b)}$ , $X_U^{(a)}$ , and $X_U^{(b)}$ ; $X_L$ is the							
	labeled set; $X_U$ is the unlabeled set.							
	<b>Output:</b> Estimated function $\bar{f}^{(a)}$ and $\bar{f}^{(b)}$ .							
1	Choose the kernel K and do KPCA on the whole data							
	set; get $\psi(\mathbf{x})$ with Eq.(6);							
2	Solve the convex optimization problem in Eq.(14) with							

quadratic programming[Gill and Wong, 2015];

3 Output  $\overline{f}^{(a)}(\mathbf{x})$  and  $\overline{f}^{(b)}(\mathbf{x})$ ;

Computation Complexity Analysis. From the algorithm flowchart we see that the main computational cost of MSMC lies in two parts: (1) to perform KPCA on the whole dataset; (2) to solve the optimization problem (14). Conventionally, the first part costs  $O(n^3)$ , where n = l + v is the total number of data points, and the second part costs  $O(P^3)$ , where P is the size of the support vectors.  $P \approx uQ$ , where u is the size of the neighborhood in the paper and Q is the size of the support vectors of the traditional SVM-2K. Practically, u = 4 is sufficient to secure a good performance.

#### **Experiments** 5

#### 5.1 Data and Parameter Setting

While MSMC is a general method for multi-view learning with limited and noisy tagging, we report the extensive evaluations in the specific application of multi-view noisy image tagging with limited labeled training samples. We compare our method with SVM, Semi-parametric SVM (Sp-SVM) [Guo et al., 2008], fuzzy SVM [Lin and de Wang, 2004], SVM-2K [Farquhar et al., 2006], Co-training, and MSS-2K [Qi *et al.*, 2012] using the noisily tagged training images combined with the plentiful untagged images to evaluate the performances of these methods.

The NUS-WIDE [Chua et al., 2009] image dataset is used in the experiments. It includes 269, 648 web images and 81 concepts which we treat as the ground truth tags. We choose the top 75 concepts whose numbers of positive examples are larger than 200 from the dataset to form the multi-label space  $\mathcal{T}$ . Hence, the dimensionality of the additional feature space of tags  $(\mathcal{L}_r)$  for each  $T_r$  is 74. For each concept, we randomly choose 260 positive examples and 260 negative examples as the training data. In the testing set, the numbers of the positive and negative examples are both  $50.\ 10\%$  examples are randomly selected from the training data to form the perfectly tagged training set, and the left 90% examples from the training data are used as the untagged training set. In the experiments, s% noise is added into both the positive and negative examples of the perfectly tagged training set to form the noisily tagged training set. We denote the 500-D bag of words feature based on SIFT descriptions as Va, and denote the 1000-D bag of text words feature which describes the text information correlated to the images provided by the dataset as Vb.

The size u of the neighborhood  $\mathcal{N}_r(I_i)$  for each  $I_i$  is defined as the count of the nearest neighbors of  $I_i$  in  $\mathcal{L}_r$ . Further, the nearest neighbors of  $I_i$  is computed with the Jaccard similarity coefficient between  $I_i$  and other samples in  $\mathcal{L}_r$ . We define R as the ratio between the weight of the loss function for instances and in the weight of the loss function for their nearest neighbors in the optimization. For MSMC,  $R = C^{(ab)}/C^{(a)\setminus(b)}$ , where  $C^{(a)} = C^{(b)}$ . In the experiments, we choose the top M largest eigenvalues and the corresponding eigenvectors from the results of KPCA to learn the parametric function for the training data. M = 0 means that we learn no parametric function for the training data.

#### 5.2 Evaluation Metric

For each tag  $T_r$ , let  $CT_r$  be the number of correctly predicted examples,  $GT_r$  be the number of the examples which actually have the tag as the ground truth, and  $PT_r$  be the number of all the predicted examples with the tag. Then the precision  $Pre_r$ , recall  $Rec_r$ , and F1 measure are defined as

$$Pre_r = \frac{CT_r}{PT_r}; \qquad Rec_r = \frac{CT_r}{GT_r};$$
  
$$F1_r = \frac{2Pre_rRec_r}{Pre_r + Rec_r} = \frac{2CT_r}{PT_r + GT_r}$$

We evaluate the performances of the methods using the standard performance measures of Macro-F1 ( $F1^a$ ) and Micro-F1 ( $F1^i$ ). Macro-F1 averages the F1 measures on the predictions of different tags; Micro-F1 computes the F1 measure on the predictions of different labels as a whole.

$$F1^{a} = \frac{1}{E} \sum_{r=1}^{E} F1_{r}; \quad F1^{i} = \frac{2\sum_{r=1}^{E} CT_{r}}{\sum_{r=1}^{E} PT_{r} + \sum_{r=1}^{E} GT_{r}}$$

#### 5.3 Performance Comparisons

In Table 1, we summarize the F1 measure of the testing set when M = 3, u = 4, R = 0.5, and s is selected as 30, 25, 20,

15, 10, and 5 in the two views combined, for SVM, Sp-SVM, fuzzy SVM, Co-training, SVM-2K, MSS-2K, and MSMC, respectively. For each method, we obtain the combined prediction with the simple average strategy on the two views' prediction. On the task of multi-view learning with limited and noisy image tagging, compared with MSMC, SVM with the noisily tagged training set considers the training set as the perfectly tagged set and mistakenly takes the incorrect tags as the perfect tags. Although Sp-SVM exploits the geometric structure of the unlabeled data, it considers the training set as the perfectly tagged set and cannot take advantage of the information from the multi-label space. Fuzzy SVM considers the training set as noisily tagged dataset, but it cannot take advantage of the information contained in multi-label space and untagged data set. Although Co-training takes advantage of the information from the untagged data, it considers the training set as the perfectly tagged set and fails to make use of the information from the multi-label space. Similarly, SVM-2K also considers the training set as the perfectly tagged set, and fails to make use of the information from the multi-label space. Further, though MSS-2K utilizes the information contained in the multi-label space by imposing more constraints between multiple view learners, it fails to take advantage of the information contained in the untagged data to make it adaptive to the environment with limited and noisy training data. Consequently, from Table 1, we see that MSMC performs better than SVM, fuzzy SVM, Co-training, SVM-2K and MSS-2K on the task of multi-view learning with limited and noisy image annotation in all the cases as the F1measures achieved by MSMC are much higher than those achieved by the competing models in all cases.

Specially, when s is selected as 30, 25, 20, MSMC performs better than the competing models significantly. It demonstrates the robustness of MSMC when the training set is seriously noisy.

#### 5.4 Sensitivity Study

We conduct a sensitivity study on the proposed MSMC algorithm. In particular, we study how the semi-parametric parameter, the neighborhood size, and the regularization parameter affect the performance of MSMC, respectively.

We describe the F1 measure for the testing set as a function of M when u = 4, R = 0.5, and s = 20 using the combination of feature Va and feature Vb for MSMC in Figure 3(a). From Figure 3(a), we observe that the F1 measures for the testing set increase when M increases, which shows that the semi-parametric regularization is helpful to further improve the performance of the classification by better exploiting the geometric structure of the marginal distribution of the data. The curves for the F1 measures of MSMC in Figure 3(a) exhibit the major effect from M = 0 to M = 1, and then are stable when M continues to increase. This is because M=1corresponds to the most discriminative component in the data distribution. It is also a well-known fact that most of the existing semi-parametric learning methods are implemented with M = 1 [Guo *et al.*, 2008].

Figure 3(b) shows the F1 measures for the testing set as a function of u when M = 3, R = 0.5, and s = 20 using the combination of feature Va and feature Vb for MSMC. From

Table 1. The I T measures for the testing set.								
	$F1^a \setminus F1^i$ using the combination of feature Va and feature Vb							
	s = 30	s = 25	s = 20	s = 15	s = 10	s = 5		
SVM	$0.6302 \langle 0.6536$	$0.6852 \langle 0.7031$	0.7279 (0.7398)	$0.7799 \backslash 0.7858$	$0.8091 \langle 0.8114$	$0.8250 \\ 0.8261$		
Sp-SVM	$0.6436 \\ 0.6664$	0.6996 (0.7163)	$0.7381 \langle 0.7487$	0.7895 (0.7953)	$0.8151 \\ 0.8172$	0.8316 (0.8327)		
Fuzzy SVM	0.6209\0.6426	0.6901 (0.7073	0.7219 (0.7325	0.7725 (0.7808	$0.8029 \\ 0.8059$	0.8220 (0.8238)		
SVM-2K	$0.6295 \ 0.6451$	0.6735 (0.6891	0.7250 (0.7332)	0.7714 (0.7762	0.8047 (0.8054)	0.8179 (0.8190		
Co-training	$0.6693 \backslash 0.6883$	0.7134 (0.7267	$0.7489 \\ 0.7596$	$0.7944 \langle 0.8003$	$0.8161 \langle 0.8182$	0.8307 (0.8326)		
MSS-2K	$0.6396 \\ 0.6600$	$0.6972 \ 0.7154$	$0.7449 \\ 0.7539$	0.7869 (0.7925)	$0.8148 \\ 0.8165$	$0.8278 \ 0.8282$		
MSMC	<b>0</b> .7368\ <b>0</b> .7600	<b>0</b> .7628\ <b>0</b> .7675	<b>0</b> .7748\ <b>0</b> .7840	<b>0</b> .7992\ <b>0</b> .8055	<b>0</b> .8285\ <b>0</b> .8305	<b>0</b> .8392\ <b>0</b> .8384		
					0.70			

Table 1: The **F**1 measures for the testing set.



Figure 3: The F1 measures for the testing set as a function of M, u, and R using the combination of feature Va and feature Vb for MSMC, respectively.

Figure 3(b), we observe that the F1 measures for the testing set increase when the size of the neighborhood for each  $N_i$ increases, which shows that it is helpful to use the nearest neighbors of each  $I_i$  in  $\mathcal{L}_r$  to further improve the performance of the classification.

We describe the F1 measure for the testing set as a function of R when M = 3, u = 4, and s = 20 using the combination of feature Va and feature Vb for MSMC in Figure 3(c). As we defined before,  $R = C^{(ab)}/C^{(a)\setminus(b)}$  represents the ratio between the weight of the loss function for their nearest neighbors and the weight of the loss function for instances in the corresponding views. We observe from Figure 3(c) that when R increases, the curves for the F1 measures of MSMC ascend, which also shows that it is helpful to use the nearest neighbors of each  $I_i$  in  $\mathcal{L}_r$  to reduce the influence of the noise in the classification.

# 6 Conclusion

In this paper, we have studied the challenging problem of multi-view learning with limited and noisy tagging and have developed a powerful discriminative model, called MSMC, that exploits both labeled and unlabeled data through a semiparametric regularization and takes advantage of the multilabel constraints into the optimization. While MSMC is a general method for multi-view learning with limited and noisy tagging, we have reported the extensive evaluations in the specific application of multi-view noisy image tagging with limited labeled training samples on a benchmark dataset. Extensive evaluations in comparison with the state-of-the-art literature demonstrate that MSMC outstands with a superior performance.

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